Overview of the TOMAGS

The Test of Mathematical Abilities for Gifted Students (TOMAGS) is a standardized, norm-referenced test designed to assess mathematical talent in children 6 through 12 years old. The TOMAGS requires students to use mathematical reasoning and problem-solving skills to understand how to communicate mathematically to solve problems. The test was designed to identify students who have talent or giftedness in mathematics and should not be used for diagnostic purposes. The Primary level is for students 6 through 9 years old; the Intermediate level is for students 9 through 12 years old. The TOMAGS can be group administered by teachers, counselors, psychologists, and other individuals. Scoring is easy, and guidelines are provided to assist the examiner in interpreting the results. Reliability and validity are strong and support its use as an identification instrument of giftedness in mathematics.

In this chapter, we discuss the National Council of Teachers of Mathematics (NCTM) standards, characteristics of children who are gifted in mathematics, assessment of talent in mathematics, and a description of the TOMAGS and its uses.

National Council of Teachers of Mathematics Standards

The NCTM standards were most important to the development of the TOMAGS. In 1989 the NCTM published the Curriculum and Evaluation Standards for School Mathematics. The standards were an attempt to specify national, professional standards for school curricula in mathematics. Thousands of members of the NCTM were involved in the drafting, reviewing, and refining of the standards during the 3 years it took to develop and subsequently publish the document (Crosswhite, 1989).

The standards are statements that can be used to judge the quality of mathematics curriculum or evaluation methods and are based on the following five goals (NCTM, 1989):

1. “Learning to value mathematics” (p. 5)
2. “Becoming confident in one’s own ability” (p. 6)
3. “Becoming a mathematical problem solver” (p. 6)
4. "Learning to communicate mathematically" (p. 6)
5. "Learning to reason mathematically" (p. 6)

These goals were developed with the intent that students become mathematically literate and develop the ability to explore, to conjecture, and to reason mathematically. There are 54 standards divided into four categories: K through 4, 5 through 8, 9 through 12, and evaluation.

The 13 curriculum standards for Grades K through 4 are (1) mathematics as problem solving, (2) mathematics as communication, (3) mathematics as reasoning, (4) mathematical connections, (5) estimation, (6) number sense and numeration, (7) concepts of whole number operations, (8) whole number computation, (9) geometry and spatial sense, (10) measurement, (11) statistics and probability, (12) fractions and decimals, and (13) patterns and relationships. The 13 standards for Grades 5 through 8 are (1) mathematics as problem solving, (2) mathematics as communication, (3) mathematics as reasoning, (4) mathematical connections, (5) number and number relationships, (6) number systems and number theory, (7) computation and estimation, (8) patterns and function, (9) algebra, (10) statistics, (11) probability, (12) geometry, and (13) measurement.

The first three curriculum standards for both Grades K through 4 and 5 through 8 address mathematics as problem solving, mathematics as communication, and mathematics as reasoning and are reflected in the TOMAGS's construction. Problem solving includes developing and applying strategies and approaches to solve problems, and verifying and interpreting results. Mathematics as communication includes reading, writing, and discussing ideas using the language of mathematics including signs, symbols, and terms of mathematics. Mathematical reasoning includes making conjectures, gathering evidence, and building an argument to support logical conclusions.

In addition to the curriculum standards, there are 14 evaluation standards. The evaluation standards stress that several aspects of assessment should receive increased attention, whereas others should receive decreased attention. For example, "focusing on a broad range of mathematical tasks and taking a holistic view of mathematics" should receive increased attention, whereas "focusing on a large number of specific and isolated skills organized by a content–behavior matrix" (NCTM, 1989, p. 191) should receive decreased attention. The TOMAGS does this very well by focusing on a broad array of mathematical tasks with many items incorporating several mathematical concepts. This is discussed in more detail in the section on content validity in Chapter 6.

The evaluation standards are further categorized into three sections: general assessment, student assessment, and program evaluation. For the purposes of the TOMAGS, the first two sections, general assessment and student assessment, are most critical. General assessment consists of three standards: alignment, multiple sources of information, and appropriate assessment methods and uses. Student assessment consists of seven standards: mathematical power, problem solving, communication, reasoning, mathematical concepts, mathematical procedures, and mathematical disposition.

The three standards in the first section, general assessment, are particularly pertinent to the development of the TOMAGS. Standard one, alignment, refers to the degree to which assessments match the curriculum's goals, objectives, and mathematical content. Standard two, multiple sources of information and uses, directs assessors to present tasks that demand different kinds of mathematical thinking. Standard three, appropriate assessment methods, requires examiners to select instruments on the basis of their use and the developmental level and maturity of the student. The general assessment section further states that when comparing general mathematical capability of a student with that of other students or a national norm, the examiner will want to use highly reliable tests designed for maximum discrimination. The TOMAGS meets all of these criteria.

Characteristics of Students Who Are Gifted in Mathematics

In developing the TOMAGS, we also considered the characteristics of gifted mathematics students. These students have many of the following abilities:

- to recognize and spontaneously formulate problems, questions, and problem-solving steps (Greene, 1981; Scruggs, Mastropieri, Monson & Jorgensen, 1985; O'Conner & Hermelin, 1979; Sternberg & Powell, 1983; Scruggs & Mastropieri, 1984);
- to distinguish between relevant and irrelevant information in novel problem-solving tasks (Marr & Sternberg, 1986);
• to see mathematical patterns and relationships (Cruikshank & Sheffield, 1992; Miller, 1990);

• to have more creative strategies for solving problems (Devall, 1983; Miller, 1990; Shore, 1986; Dover & Shore, 1991);

• to think abstractly and reason analytically (Cruikshank & Sheffield, 1992; Marr & Sternberg, 1986; Miller, 1990);

• to be more flexible in handling and organizing data (Cruikshank & Sheffield, 1992; Devall, 1983; Greenses, 1981; Miller, 1990; Shore, 1986; Dover & Shore, 1991);

• to offer original interpretations (Greenses, 1981);

• to transfer ideas generalized from one mathematical situation to another (Cruikshank & Sheffield, 1992; Greenses, 1981; Miller, 1990);

• to be intensely curious about numeric information (Cruikshank & Sheffield, 1992; Miller, 1990)

• to quickly learn and understand mathematical ideas (Dover & Shore, 1991; Miller, 1990);

• to reflect and take a longer amount of time when solving complex problems or those with several solutions (Davidson & Sternberg, 1984; Miechenbaum, 1980; Sternberg, 1982; Wong, 1982; Woodrum, 1975); and

• to persist in finding the solution to problems (Ashley, 1973; House, 1987).

While each mathematically talented student may not have all of these abilities, researchers agree that these students do not necessarily have the ability to compute. Miller's (1990) definition emphasizes this point by stating that mathematical talent "refers to an unusually high ability to understand mathematical ideas and to reason mathematically, rather than just a high ability to do arithmetic computations." (p. 2)

Grades and traditional achievement tests may not identify gifted students with mathematical abilities. Mathematics classes frequently focus on computational accuracy rather than problem solving, so children who achieve high grades in school may not be mathematically talented. Similarly, standardized achievement tests concentrate on low-level tasks and also may not identify gifted students (Romberg & Wilson, 1992). A need exists to assess problem-solving and reasoning abilities of students who are gifted in mathematics. The TOMAGS was developed to meet this need.

Assessment of Giftedness in Mathematics

Assessment of student knowledge of mathematics is carried out for many purposes. The TOMAGS was developed to identify giftedness in mathematics. The most critical feature of this type of assessment is the degree to which it discriminates among students who have varying degrees of talent in mathematics.

Over the last two decades there has been a substantial increase in identifying children with mathematical talent. The roots of this movement can be traced back to 1971, when Julian C. Stanley started the Study of Mathematically Precocious Youth (SMPY) at Johns Hopkins University (Stanley, 1991). Stanley used above-level testing because most age-appropriate tests were not difficult enough for this population (see the next section for a more thorough discussion of this concept). He used as his original criterion for entry into the program a score of 500 to 800 on the Scholastic Abilities Test—Mathematics (SAT–M) for students who were younger than 13 years old. In 1979 a unit was developed at Johns Hopkins, independent from SMPY, to conduct talent searches and academic programs. The creation of this unit enabled SMPY to raise its lower score from 500 to 700 on the SAT–M and concentrate its efforts on students scoring from 700 to 800.

Today many programs conduct talent searches in mathematics. For example, a similar program is the Elementary Student Talent Search conducted at Carnegie Mellon University (Lupkowski-Shoplik & Kuhnle, 1995). These talent searches have one thing in common: All use above-age testing when identifying mathematical talent. As mentioned earlier, above-level testing is used because grade-level tests lack difficult items. Whereas above-level testing appears to have validity with many children, it has been used primarily with children in Grade 4 or above. When assessing younger children, NCTM stresses the use of developmentally appropriate instruments, which makes above-level testing potentially problematic.

In recent years, educators have begun assessing for mathematical talent using alternative methods in addition to above-level testing. Sheffield (1994) discussed several important methods that are essential in the assessment of talent in mathematics. Although many of these, such as student interviews, are not suitable for inclusion in a standardized test, one method discussed is using open-ended questions. The TOMAGS uses an open-ended question format with developmentally appropriate problems.
Needs in Identifying Gifted Children

Stanley (1976) pointed out that tests generally used to identify gifted children were inappropriate because they failed to have enough “ceiling.” Grade- and age-appropriate tests are often too easy for gifted children. Testing the child’s limits can be achieved only if the test is difficult enough to determine the extent of the child’s knowledge. For example, a second-grade child capable of sixth-grade math work is not going to show his or her true abilities in mathematics by taking a second-grade achievement test. In addition, two students who earn the top score on the test may, in fact, have very different abilities in mathematics if the test is too easy. In this case, the test does not accurately measure what students are able to do (i.e., it does not discriminate among students with high ability). In addition, if a test does not have enough difficult items, a student missing only one item may receive a percentile rank that eliminates him or her from entry into the gifted mathematics program when, in fact, the student has high ability in mathematics. Unfortunately, most tests used to identify students as gifted are not developed for use with gifted children, but for use with heterogeneous groups of children.

The TOMAGS was constructed specifically for children gifted in mathematics. The two levels of the test—Primary and Intermediate—have many difficult items that allow examiners to test the limits of gifted children.

We conclude these three introductory sections on basic concepts by referring our readers to the content validity section in Chapter 6. In that section, qualitative and quantitative evidence is presented that supports the alignment of the NCTM standards with the abilities that are measured by the TOMAGS and the formats in which the abilities are tested.

Description of the TOMAGS

The TOMAGS was developed to address some of the concerns presented in the previous sections and to identify children who demonstrate giftedness in mathematics. The TOMAGS consists of open-ended questions presented in a problem-solving format. The TOMAGS Primary consists of 39 problems and the TOMAGS Intermediate consists of 47 problems. Campbell and Bamberger (1990) defined mathematical problem solving as envisioned in the NCTM standards as “students actively involved in constructing mathematics . . . [and] apply[ing] new mathematical knowledge” (p. 15). The TOMAGS was designed to estimate a child’s capacity to use mathematical knowledge in novel situations and, in some cases, to construct new strategies to solve a problem.

The TOMAGS requires the child to work through a series of problems that are aligned to the NCTM standards. The test examines a sample of the child’s ability to use flexibility in mathematical reasoning and transfer already learned mathematical knowledge to new situations. The test problems use, in some cases, knowledge taught formally in school and, in other cases, knowledge not taught formally in school.

Uses of the TOMAGS

One of the major purposes for administering the TOMAGS is to obtain information that is helpful in identifying children for gifted classes that emphasize mathematics. It is not intended for identifying children for classes emphasizing talent in leadership, visual or performing arts, and/or academic areas other than mathematics.

Second, the TOMAGS may be used as a screening instrument with the entire pool of students being considered for the gifted mathematics program or as a second-level screening instrument with only the nominated students. The TOMAGS provides norms for both gifted and normal groups. “Borderline” children may be more accurately identified using the gifted norms.

Third, the TOMAGS may be used clinically to examine a child’s relative strengths and weaknesses in the individual constructs incorporated into the test. In this way, a child’s potential is not hidden through the quantification of multiple test scores.

Fourth, the TOMAGS has value as a research tool, especially for researchers who need standardized instruments to study talent in mathematics. Its results can be used to evaluate the effectiveness of a gifted mathematics program or to test various theories of mathematical talent.