a mathematics unit for high-ability learners in grades 2–4

# Spatial Reasoning

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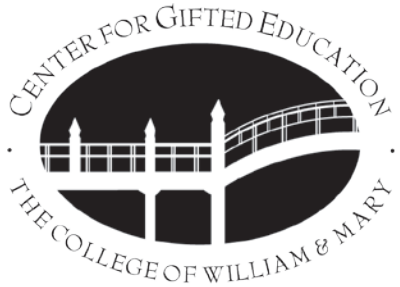


# Contents

## *Part One: Introduction*



## *Part Two: Lesson Plans*



## *Part Three: Unit Extensions and Resources*

# Lesson 2: Introduction to Dimensions

## *Instructional Purpose*

- To introduce the idea of dimensions
- To demonstrate the advantage of information gained by larger numbers of dimensions available to our perceptions
- To build geometric solids and identify the 0-, 1-, and 2-dimensional characteristics of the solid

## *Materials and Handouts*

- Frogs (one frog per student; Handout 2A)
- Model of Dimensions (Teacher Resource 1)
- Dimensions (Handout 2B)
- Dimensions Answer Key (Teacher Resource 2)
- 1-inch square tiles or paper squares (about six per student)
- A pop-up children's book
- Toothpicks and gumdrops, marshmallows, clay or Play dough, or commercially available materials for building polyhedra
- One small cube (1-inch cube or larger is best) for each student

## *Vocabulary*

**Line:** A set of points that form a straight path extending infinitely in two directions. Lines often are called *straight lines* to distinguish them from curves, which are often called *curved lines*. Part of a line with two endpoints is called a *line segment*.

**Plane:** A flat surface containing all of the straight lines that connect any two points on it.

**Point:** A location in space.

**Polyhedron:** A solid figure bounded by flat faces. Plural is *polyhedra* or *polyhedrons*.

**Prism:** A solid figure having bases or ends that are parallel, congruent polygons, and sides that are rectangles or other parallelograms.

**Vertex:** A corner point of a geometric figure. The plural is *vertices*. In a 2-dimensional figure, it is the point where two line segments meet. In a 3-dimensional figure, it is the point where 3 or more faces meet.

## *Activities*

1. Introduce the idea of dimensions by drawing a long line across the chalkboard. Welcome students to Lineland and give each student small paper **Frogs (Handout 2A)**. Ask students to imagine that one of the frogs lives in Lineland. He cannot go anywhere else except move along the line. Have a student come to the board and demonstrate how the frog would move in Lineland. (Forward and backward; no jumping off the board or moving off the line).

2. Have frogs move to Flatland by using a tabletop in the classroom. Use the following questions to have a discussion with students.
    - How do the frogs move in Flatland? (No jumping, but they may move forward, backward, or sideways.)

- What do the paper frogs of Flatland look like when they see each other? Encourage students to stoop so their eyes are level with the tabletop. (The frogs look like a line segment as only the edge of the paper can be seen by other frogs in this flat environment.)
- What are advantages of Flatland over Lineland? (More directions to move.) What are the constraints? (They cannot jump over each other; only sliding around is permitted.)

3. Move the frogs to Spaceland, which is represented by the classroom. Discuss the similarities of the different lands and the advantages of Spaceland. In all of the lands, the frogs can move forward or backward. In Flatland, they can get around each other. But, they have the most mobility in Spaceland where they can jump up into the air and over each other, in any direction.

4. Introduce the term *dimension* using the following definitions and connect back to the three lands.
    - **0-D:** A point is a location in space. It has no dimensions.
    - **1-D:** A line is a set of points that form a straight path extending infinitely in two directions. A line segment is a part of a line with two endpoints. In everyday speech, lines often are called straight lines to distinguish them from curves, which are often called curved lines. In this unit, a line means a straight line.
    - **2-D:** A plane is a flat surface extending infinitely in all directions. All 2-D objects lie in a plane. For example, a rectangle is a two-dimensional figure. Two numbers (generally called length and width) are required to determine it.
    - **3-D:** Space. Movement may occur in all directions. A cube is a three-dimensional figure. Three numbers (generally length, width, and height) are necessary to describe its dimensions.

5. Use the projection screen in your classroom as an example of the different dimensions. Indicate that the roller represents a line; that is a 1-D figure. Pull down the screen and ask students how many dimensions the screen represents. (2-D.) Tell students that if you could create a box popping out from the screen, it would represent a 3-D object.

6. Use parked cars as another example of the different dimensions. A single car can be thought of as a large point (0-D). When cars parallel park next to a curb along the street, you can think of this as one-dimensional parking (points extended in one direction). A parking lot can be thought of as a 2-D object (points extended over a plane). A parking garage is a 3-D parking arrangement because it extends the 2-D parking lot in one more direction.

7. Ask students to find objects in the room that are examples of 1-D, 2-D, and 3-D items. For example, the line between tiles on the floor (1-D), the tiles on the floor (2-D), and a desk (3-D).

8. Give students 1-inch tiles. Tell students to think of them as very large points. On the overhead projector, place one tile and add to it in one direction. Have them copy your line of tiles at their desks. Ask students in how many directions are they adding tiles. (One.) Explain to students that this represents one dimension.

9. Make a rectangle of tiles on the overhead projector and have students do the same. Ask students how many dimensions are represented. (Two.)

10. Have students make a 2 x 3 rectangle with tiles and then build upward on the base. Ask students how many dimensions are represented. (Three.)

11. Show students the **Model of Dimensions (Teacher Resource 1)**. Use the point to represent zero dimensions, the line to illustrate one dimension, the rectangle to illustrate two dimensions, and the pop-up figure to illustrate three dimensions.

12. Show students a pop-up book and discuss why this technique is effective in a storybook. Use the following questions to lead a class discussion. Time permitting, you may want to let students design and make their own pop-up pictures.
    - Why does the author use the pop-up image?
    - Would you prefer to read a pop-up version of a picture book or a version with only pictures drawn on the 2-D pages? Why?
    - When you see 2-D pictures in a book, can you imagine the 3-D objects that are shown?
    - Why do you think a 2-D picture is sufficient to make us understand what the 3-D object is?

13. Distribute materials to create models with Zometools, toothpicks and gumdrops, marshmallows, clay, or play dough. Have students make a triangle. Ask students how many dimensions the triangle has. Have students make a second triangle the same size and shape as the first. Then they should connect the two triangles to make a prism (see Figure 1). Identify vertices (corners), edges, faces, and the polyhedron. Ask students how many dimensions are needed for each of these characteristics. (0, 1, 2, and 3.) Tell students that a solid figure bounded by flat faces is called a polyhedron.

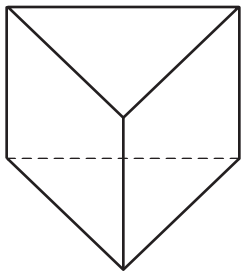


**Figure 1.** Example of polyhedron.

14. Use the following questions to have students discuss in pairs and then as a whole group. Encourage students to ask their parents if they can look at the map next time they go on a car trip; this is a good spatial activity.
    - How many dimensions does a map have?
    - Why do you think maps are not made to be 3-D?
    - Has anyone had the experience of riding in the car in a hilly area where you were surprised because the map made the road look flat?

15. Distribute **Dimensions (Handout 2B)** and have students complete it. Assess student responses with the **Dimensions Answer Key (Teacher Resource 2)**.

16. Close the lesson by reminding students that this mathematics unit is about spatial reasoning, which is the ability to understand movement or change in images or objects within any number of dimensions or movement between dimensions.

## *Notes to Teacher*

1. This unit represents a point with a dot so that the point is visible, but the point has no length, width, or height. Students may argue that points have very minute length and width measurements. However, teachers need to reinforce the idea that it is not the representation, but the definition that determines what a point is. A point is a location in space, and has no dimensions.

2. Euclid, a Greek mathematician who lived about 325 B.C.–265 B.C. and is often called the father of geometry, gave the vague definition of a point as "that which has no part."

3. It is difficult to say that any real-world object is one-dimensional, as the width of the object is greater than a point. Even a line drawn on paper has width greater than a point. But, we think of items such as spaghetti as being one-dimensional objects as the width is negligible. The distinction between two and three dimensions is the most relevant for functioning in the real world. Therefore, this unit will concentrate on two and three dimensions and movement between those two dimensions.

4. You may use commercially available materials such as Zome products or D-stix, to build the three-dimensional solids in this lesson.

5. Students may have trouble with the idea that a vertex of a polyhedron is a point. Teachers can demonstrate this concept if they use marshmallows and toothpicks as building materials. The marshmallows are used as the vertices of polygons and polyhedra. Whenever you join two toothpicks, the intersection is a point and is represented by a marshmallow.

6. Be aware of student confusion of the following terms in this unit. When using each term in context, it helps to write the word on the board so that students get a visual cue for the distinctions in the words.
    - **polygon:** 2-D figure,
    - **polyhedron:** a 3-D figure such as those that were constructed in this lesson, and
    - **Polydrons™:** commercially distributed plastic polygons that snap together to build polyhedrons. (List this word if only you are using these materials.)

7. For an elaborate example of a pop-up book, look for one illustrated by Robert Sabuda. One example is *Alice's Adventures in Wonderland*.

8. Food items such as M&M's, Cheerios, and raisins, can be used instead of tiles to illustrate dimensions. However, be aware that use of candy can distract students from the substance of the concepts. You can reinforce the ideas of dimensions by talking about M&M's as being big points and therefore 0-D, licorice strings as a 1-D candy, Fruit Roll-ups as 2-D, and a cake as 3-D (especially if it is a rectangular cake, you can measure the three dimensions: length, width, and height).

## *Assessment*

- Dimensions (Handout 2B)

## *Extensions*

The following student activities can be used to extend the lesson.

1. Have students build any polyhedron they like with Polydrons™. (See the Unit Materials section in the Introduction to the Unit for a recommended source for purchasing these plastic materials.) Have each child present his or her structure with an explanation of how many vertices, edges, and faces it has. Take digital photos of their creations. Ask students how many dimensions the picture has and how many dimensions are represented in the complete structure.

2. Have students make a pattern with colored tiles by adding tiles in the direction of one dimension. They should use three colors and have a classmate continue the pattern.

3. Ask each student to make a pattern with colored tiles in two dimensions using three colors and have a classmate continue the pattern.

4. Have students make a pattern with colored cubes in three dimensions. Have them use two colors and have a classmate continue the pattern.

5. Read *Flat Stanley* by Jeff Brown and Scott Nash. Stanley experiences life as a two-dimensional figure in a three-dimensional world. Discuss the advantages and disadvantages of being 3-D or 2-D in Stanley's situation.

6. Bring in a 3-D Tic-Tac-Toe game. Have students play and compare it to the usual two-dimensional version.

7. Have students act as points. Tell them to imagine that they are a point living in 0 dimensions. Have students demonstrate how they can move in 0 dimensions. Repeat for 1, 2, and 3 dimensions.

# Frogs (Handout 2A)

Name_____ Date_____

# Model of Dimensions (Teacher Resource 1)

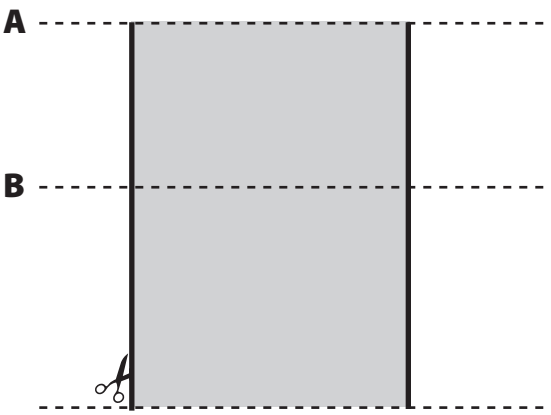
1.

- How many dimensions? _____

How many dimensions? _____

How many dimensions? _____

2.




Make a valley fold along line A.

Make a mountain fold on this line, cut on the 2 solid lines, pop up the shaded rectangle, and reverse the crease on the white part of the paper along line B to make the dotted line into a valley fold.

Make a valley fold on this line.

How many dimensions are represented by the pop-up figure? _____

Name_____ Date_____

# Dimensions (Handout 2B)

1. How many dimensions are represented by each of these?

   a. A crack in the sidewalk where two squares are joined _____

   b. A picture of a car _____

   c. A paper cup _____

2. Your teacher will give you a cube to use to answer these questions.

   a. How many vertices (corners) does the cube have? _____

      How many dimensions does the corner represent? _____

   b. How many edges does the cube have? _____

      How many dimensions are represented by an edge? _____

   c. How many square faces does the cube have? _____

      How many dimensions does a square face represent? _____

   d. Circle the best answer. A cube is an example of:
      i. A one-dimensional object
      ii. A two-dimensional object
      iii. A three-dimensional object