a mathematics unit for high-ability learners in grades 6–8

Moving Through **Dimensions**

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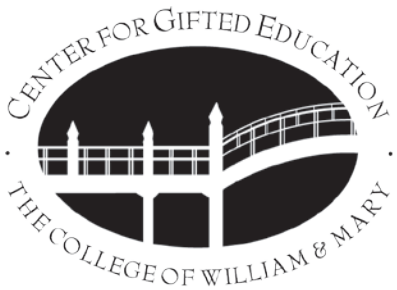

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Lesson 4: Projecting Solids Into Two Dimensions

Instructional Purpose

- To project three-dimensional objects into two dimensions
- To record the projection using a mat plan

Materials and Handouts

- Mat Plan Example (Handout 4A)
- Mat Plans (Handout 4B)
- Mat Plans Answer Key (Teacher Resource 1)
- Different Views (Handout 4C)
- Different Views Answer Key (Teacher Resource 2)
- Set of wooden geometric solids
- Objects for projecting (e.g., JELL-O box, coffee mug, black binder clip, soup can, penny, spool of thread, stapler, tape dispenser, apple)
- Math journals

Activities

Part I

1. Tape newspaper to the front of the overhead projector so that students cannot see what you are setting on the glass. Put a clean transparency sheet on the glass to protect it. Show students a rectangular prism (with the length, width, and height of all different measurements, such as a chalk box) by setting it on the center of the projector glass. Tell students that this is the top view. Turn it to its side and tell students that this is the side view. Turn it so it rests on the front and tell students that this is the front view. Ask students to identify the object.

2. Tell students that the shadow or image of the solid on the screen is called a projection because you are projecting the image. Explain that they are looking at a two-dimensional image of a three-dimensional object. Ask students how many different projections there are for a given object. (Three, there is a projection for each dimension. Use the top view, side view, and front view to identify the three views.)

3. Put a hemisphere, from a set of geometric solids, on the overhead projector sitting it on its circular base. Ask students to identify what possible geometric solids it might be. (Sphere, cylinder, cone, and hemisphere) Show students the side view and have them predict the front view. Show students the front view (which in this case is the same as the side view).

4. Repeat with a cone and pyramid. If you use a sphere, turn off the overhead projector and put a small piece of double-sided tape on the sphere. Tape it to the transparency so it does not roll. Then turn on the overhead.

5. Project some everyday objects on the overhead. Ask students how many views they need to see before they can identify the object. (Sometimes you might know

in two projections because the third view is the same as one of the others. For example, a cylinder has the same front view and side view.)

6. Put an object such as a book on the overhead. Show one view and ask students to identify the object. Share a second view, and then a third. Students may not know for sure that it is a book, but they should know it has the shape of a rectangular prism.

7. As with the sphere, turn off the overhead. Put a small piece of double-sided tape on the barrel of the mug on the opposite side of the handle. When you stick it to the transparency, the handle should be aimed upward and therefore not visible in the projection, as shown in Figure 4.

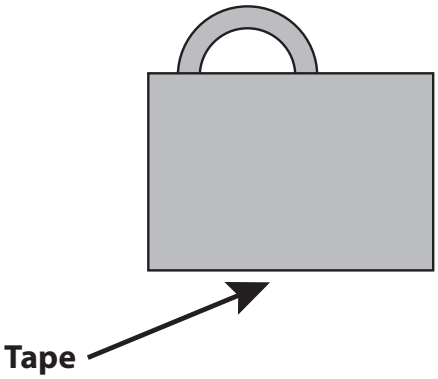


Figure 4. First orientation of the coffee cup.

8. Turn on the overhead. Tell students that this is the side view and ask what the object might be. Students will not know as the projection should be similar to a rectangle. Turn the mug on its side so students can see the front view that looks something like Figure 5.

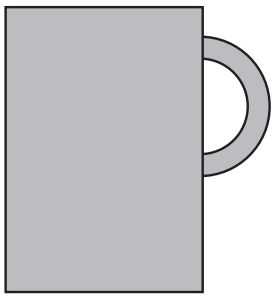


Figure 5. Second orientation of coffee cup.

9. Have students draw what they think the top view looks like in their math journal or notebook. It should be something like Figure 6.

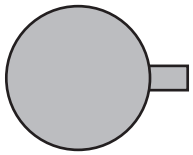


Figure 6. Top view of coffee cup.

10. Check by putting the mug on the overhead projector or have students stand over the cup and look directly down into it. Hint: If you cover one eye you get a flatter image.

11. Project other objects such as a JELL-O box, binder clip, soup can, square-based box, pencil, square pyramid, cube, and sphere. Have students classify the objects according to their projections using the following categories:
    a. All three views different (JELL-O box, binder clip)
    b. Two views the same (soup can, square-based box, pencil, square pyramid)
    c. All three views the same (cube and sphere)

12. Have students build cube structures and draw projections on grid paper: top view, front view, and side view. Encourage students to look directly at the structure with one eye covered or closed. This takes away the depth perception and makes the image look two-dimensional. Ask students what can be gained by seeing all three projections.

13. Give students a mat plan (you can copy one from this book or make your own). Ask them to draw the front view, top view, and side view without actually building the structure with cubes.

14. Discuss that these two-dimensional images of three-dimensional objects are created by projecting top, front, and side views called *orthogonal projections* in mathematics. Have students look for the term *orthogonal* in a dictionary and report on their findings.

15. Tell students that shadows are projections. If you can darken the room enough to make a clear shadow, use a good flashlight and hold it about 4 to 5 feet away from an object and view the shadow on the wall. Shine the light from the top, side, and front views of the object. The effect is similar to that of using an overhead projector to project shadows.

Part II

1. Distribute Mat Plan Example (Handout 4A) and display a transparency. Ask students how many cubes are needed to build the structure and how they know (12). Tell students that one way to record information about structures they build is to make a mat plan. A mat plan is a floor plan of the base of the structure with the number of cubes placed on each square of the plan. Ask student to compare the mat plan to the top view projection of an object. (The same in shape but the projection has no numbers written on it.)

2. Distribute Mat Plans (Handout 4B) and have students complete the handout. Tell students if they build the structure as shown, they should rotate it 45 degrees to arrive at the traditional orientation for the mat plan.

3. Have students work in pairs with a divider between them so they cannot see their partner's structure. Have one student build a cube structure and keep it hidden from his or her partner. Have the student create a mat plan for the structure and pass the mat plan to his or her partner. The partner should build the structure following the mat plan. Remove the barrier to compare the structures.

4. Distribute Different Views (Handout 4C) and have students complete it. For extra challenge, have them complete it without using cubes.

5. Encourage students to bring in objects to project on the overhead projector. This will prompt them to look at everyday objects with respect to the nature of their projections. They can use a flashlight at home to try various projections.

Notes to Teacher

1. If you have a glass table at home, you can practice projections by placing objects on the table and viewing them from underneath. You may suggest this activity to students to try if they have a glass-topped table at home.

2. The projection of the right side view of a cube structure often is different from the projection of the left side view. If the directions say, "Side view," it means "Right side view."

Assessment

- Mat Plans (Handout 4B)
- Different Views (Handout 4C)

Extensions

1. Go to this Web site (http://www.learner.org/teacherslab/math/geometry/space/plotplan/index.html) and see if you can make a mat plan that is correct for the given front and right side projections. This Web site uses the term *plot plan* instead of mat plan.

2. Play the game Point Out the View, in which characters positioned around a cube structure need to show what the structure looks like to them. The game can be found at http://pbskids.org/cyberchase/games/pointofview/pointofview.html.

3. Describe in your math journal what a projection from two dimensions to one dimension would look like. Hint: Think of a flashlight in Flatland shining from above the triangle in Figure 7 and projecting a shadow onto the x-axis. The shadow is one-dimensional because it only has length.

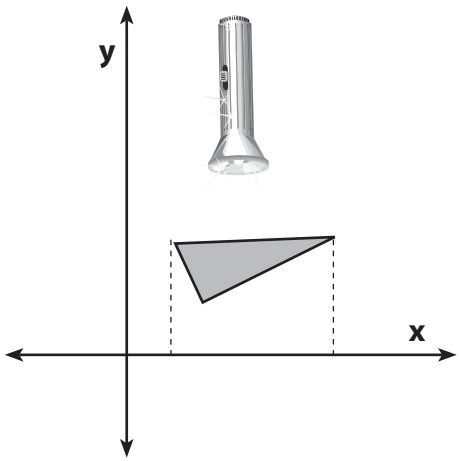


Figure 7. Flashlight projection above triangle.

4. Then repeat by shining the light from the right of this triangle (see Figure 8) and projecting a shadow onto the y-axis. The shadow is one-dimensional because it only has length.

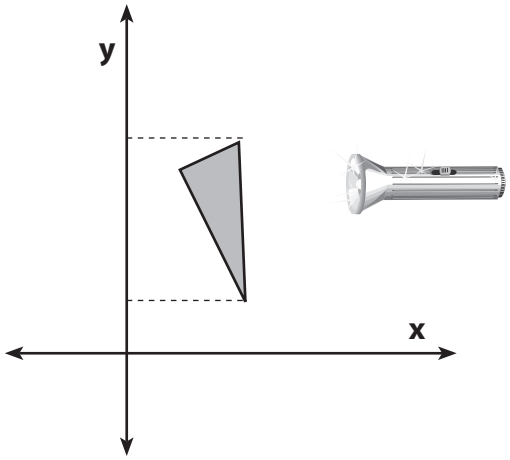


Figure 8. Flashlight projection to side of triangle.

5. Use activities from pages 41–51 from the book *Spatial Problem Solving With Cuisenaire Rods*. This set of blackline masters has a challenging section on making drawings (top, front, side views) of structures made from Cuisenaire Rods.

6. Explore http://www.fi.uu.nl/toepassingen/00200/toepassing_wisweb.en.html, which is a three-dimensional object viewer. It shows a drawing of a three-dimensional objects in four different views: front, left, right, and top. Choose some objects to view and then draw the views before the applet shows them.

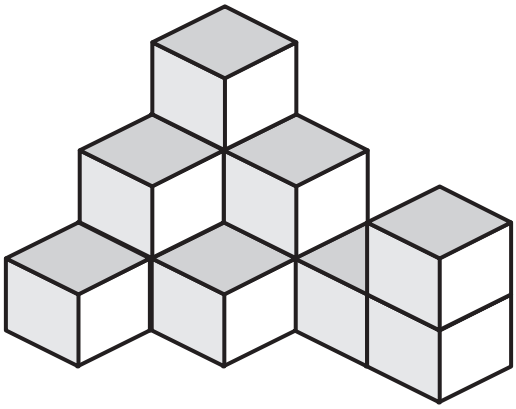
7. Create a set of projections for a cube structure such that there is more than one solution (i.e., more than one structure has these projections). Give the mat plan of your multiple solutions.

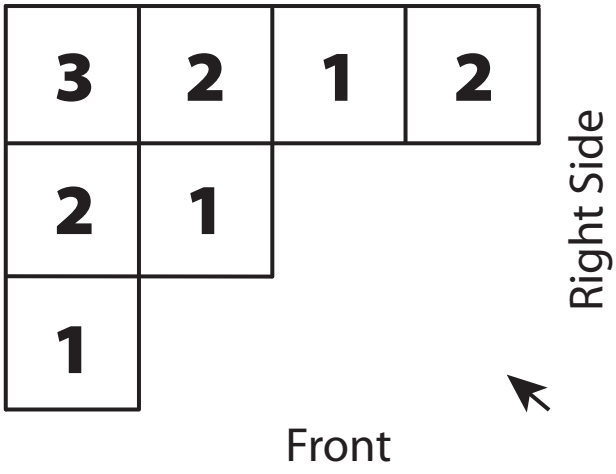
Resources

- *Connected Mathematics: Ruins of Montarek* by Glenda Lappan: This unit from the Connected Mathematics project at Michigan State University does many of the projection type of problems done in this lesson. It involves creating and interpreting architectural and isometric representations.
- *Spatial Problem Solving With Cuisenaire Rods* by Patricia S. Davidson and Robert E. Willcutt: This set of blackline masters has a challenging section on making drawings (top, front, side views) of structures made from Cuisenaire Rods.
- "Developing Spatial Sense: Comparing Appearance to Reality" by Gwen Kelly, Tim Ewers, and Lanna Proctor in *The Mathematics Teacher*, December 2002, pp. 702–712: This article has eight pages containing 16 task cards involving various top, front, and side view activities.
- *Spatial Visualization Unit* (Middle Grades Math Project), by Mary Jean Winter, Glenda Lappan, Elizabeth Phillips, and William Fitzgerald: This is an extensive unit that deals with isometric drawings and mat plans.

Mat Plan Example (Handout 4A)

Directions: Use the structure's image below to help you visualize how mat plans show how many cubes are in each tower of a structure. The figure at the top is the view of the cube structure as it looks from the corner marked with the arrow in the bottom figure.





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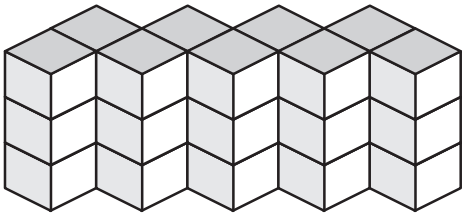
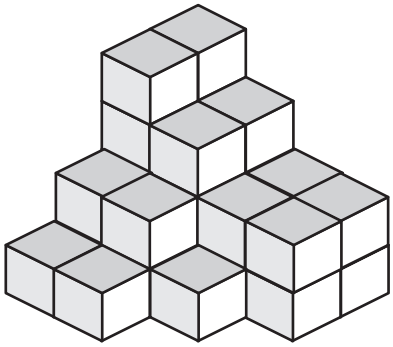
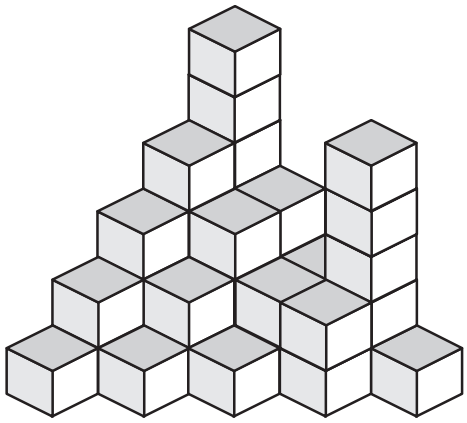
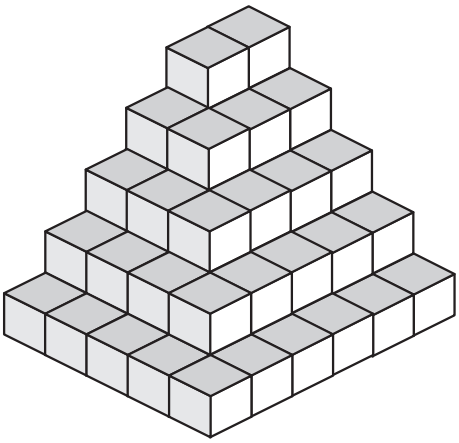
Mat Plans (Handout 4B)

Directions: How many cubes does it take to build each of these? Make a mat plan for each structure.

	Guess: Build and Check:	Make a mat plan:
	Guess: Build and Check:	Make a mat plan:

	Guess: Build and check:	Make a mat plan:
	Guess: Build and check:	Make a mat plan: