

SECOND EDITION

# DEVELOPING MATH TALENT

A COMPREHENSIVE GUIDE TO  
MATH EDUCATION FOR GIFTED  
STUDENTS IN ELEMENTARY  
AND MIDDLE SCHOOL

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&  
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## CHAPTER 9

# Case Studies

### KEY POINTS

- Mathematically talented students can be identified at a young age, and their parents are often the first to recognize their talents. Parents play a critical role in advocating for appropriate challenges for their children.
- Objective information is essential in helping parents to be effective advocates for their children.
- The Diagnostic Testing→Prescriptive Instruction model helps ensure that exceptionally mathematically talented students study mathematics at a steady rate.
- Long-term planning is essential so that students are always studying an appropriately challenging level of mathematics.
- Mathematically talented students benefit from finding an intellectual peer group; this is important for both boys and girls.

*All parents worry about their child's social-emotional development and Zach's were no exception. Now that their highly gifted son is in graduate school, they have the luxury of knowing that he has grown into an individual who, as described by his dad, is a "good citizen." Teachers and other parents may find it useful to know that Zach's father believes that, if he had it to do all over again, he "would have worried less about the social aspects of Zach's development."*

We have worked with thousands of students over the past several decades. From this work, we have identified case studies that highlight the main points presented throughout this book. We are grateful to the families and students who so willingly shared their experiences and

insights with us in order to help other mathematically gifted students, their families, and their educators.

The case-study approach is extremely useful for the families with whom we work as well as in our own research. The stories we relate in this chapter help to illustrate the points we have made in earlier chapters. Although case studies are anecdotal in nature, they offer a general model that can apply to individual students. Both parents and teachers have told us that they like reading case studies because they realize that others have blazed the trail, and they do not have to “go it alone.” We think our case studies are somewhat unique because they integrate specific individual information with a robust theoretical approach that has been proven to be effective with hundreds of thousands of students (see Chapter 5).

### **Rosie: It’s April, and It Is Time to Plan for Next Year<sup>1</sup>**

Rosie is a cute 7-year-old who loves taking gymnastics and swinging on her swing in the back yard. She skipped several grades and is currently in third grade. Her parents first contacted us in October, when they were thinking of having her take the EXPLORE test. Her father gave some examples of the math they were doing together at the time:

Recently, we’ve been plotting simple functions at home like  $y = x^2$ . She can solve equations like  $3x^2 - 25 = 50$ , and do the basic operations on fractions and decimals. We’re trying to convince the school that she can do more than fourth-grade math (the current accommodation beyond her grade skip), but she makes so many mistakes on the pretests that they can’t tell the difference (she makes errors in execution, doesn’t read the problem properly, and sometimes even leaves items blank).

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<sup>1</sup> Most of this case study was written by Rosie’s dad. We thank him for his comprehensive notes and attention to detail.

Rosie demonstrated her intellectual abilities at a young age. She read her first sight word on a billboard advertisement for the local supermarket at 18 months, although her parents didn't think much of it at the time. Rosie's speech was so advanced by age 2 ½ that her father, a software developer turned stay-at-home dad, decided to research early reading. He found that some children had begun reading at her age, and so he set about trying to instruct her. After a few lessons in a formal system, she let him know she was not interested by throwing the book across the living room. Her father decided to try again 6 months later when she turned 3. Rosie negotiated a reward of a cookie-milkshake for completing the remainder of the instructional book, which she finished in 6 months.

At this point, Rosie could read second-grade books. She attended a Montessori preschool and demonstrated that not only could she count objects (a skill she had acquired at a young age), but she also was ready for addition, subtraction, and counting by twos, threes, fours, and all the digits through 10. She also appeared to understand the basics of algebra (which her father explained as a letter "pretending to be" a number) and could solve simple equations like  $5 + b = 10$ . One day after preschool, she said, "I told my friend Caitlyn that a letter could pretend to be a number." Her father asked, "What did she say then?" "She just looked at me weird," Rosie replied.

Rosie was developing a comprehension of place values through thousands by the time she finished her second year of preschool at age 4 ½. The following summer she was tested at a university school psychology program, which her parents elected to use as a trial before investing a lot of money in testing. Rosie scored a Full Scale 141 on the Stanford Binet V, with a Nonverbal IQ of 144 and a Verbal IQ of 135. The university tester made a comment to Rosie's parents that that, in the experience of their school psychology program, they had "never seen anything like her." The written report recommended that, "She may be considered for placement in first grade." For Rosie's parents, this was the first professional call for radical acceleration.

Rather than send her to school for kindergarten at the age of 4, she was homeschooled with her father as her teacher. In that year, she read more than 15,000 pages of chapter books, completed eight spelling

workbooks, performed science experiments on nearly a daily basis, completed a world history course, and stepped through second- and third-grade Singapore math as well as *Daily Word Problems (DWP): Grade 3* and part of *DWP: Grade 4*.

Her parents had heard of the Davidson Young Scholars program (<http://www.ditd.org>) and thought that membership would benefit both Rosie and themselves. With only a four-point gap between Rosie's IQ score and the Davidson cutoff, they decided to have Rosie retested by someone experienced with high-achieving girls, Dr. Sylvia Rimm, at her Family Achievement Clinic in Cleveland, OH. Rosie's WISC-IV results placed her IQ at "146+," with a verbal Comprehension of "155+." Dr. Rimm suggested further testing to determine Rosie's "actual" IQ, but her parents declined, feeling that it would provide no additional information that would impact their decisions. Rosie was accepted as a Davidson Young Scholar. Her father describes the Davidson resources as "invaluable" and would encourage the parents of any highly advanced child to apply.

Dr. Rimm recommended that the family place Rosie in a typical school setting, with at least a double grade skip, for Rosie to have an opportunity to work with peers and exchange ideas. Although opposed to the plan, her father agreed to proceed on a trial basis. The reaction from the two leading local private schools was, "We don't accelerate." One local parochial school did not return the parents' call. The principal of the leading local parochial school ridiculed them with the response, "What does she do, suck her thumb and read a book?" Similarly, the principal of that school implied that Rosie's parents were lying about her abilities, while he boasted that his school's curriculum was 6 months ahead of the public school (little comfort to a child 4 or more years ahead). Although Rosie's father had been openly critical of the local public school's gifted services and felt that he had burned bridges, Dr. Rimm encouraged him to try there. Initial opposition to the idea was high, and to make matters worse, less than 2 weeks remained before the start of the school year. However, the district agreed to allow Rosie to go directly into second grade after its first-time use of the *Iowa Acceleration Scale* and a conference call with Dr. Rimm. This was a double skip based on Rosie's October birthday. The administration

wisely consulted the second-grade teachers to see who was most receptive to instructing Rosie; luckily, someone stepped forward to meet the challenge.

In Rosie's state, Individualized Education Programs (IEPs) for gifted students are required by regulation, but there is no enforcement behind the regulations. In fairness to the school, her parents report that the entire year was an experiment to answer the question, "What would happen if this student went to regular school but skipped two grades ahead?" Rosie's IEP focused mostly on her integrating into to school life in second grade, but there were other accommodations. Rosie was placed in the highest guided reading group in her grade (which read fourth-grade books) and was given access to many fifth-grade books for pleasure reading (her teacher formerly taught fifth grade). Rosie received a specialized "challenge" spelling list, which her teacher eventually shared with all four second-grade classes. Although allowed to work ahead on math speed tests as she was able, this was an inconsequential accommodation as Rosie was not a rapid calculator. Similarly, she was allowed to work ahead in the district's computerized adaptive math curriculum (Compass), but she made so many computational mistakes that the system never caught up with her conceptual knowledge. Perhaps most helpful, Rosie engaged in weekly pull-out sessions for math problem solving with other advanced students. Rosie continued her work at home on *DWP: Grade 4*, as well as supplementing it with the HeyMath! website (<http://www.heymath.com>).

Rosie was allowed to bring various extension projects into school. For the 100th Day of School celebration, in which the number 100 was discussed, Rosie and her father taped together 20 sheets of graph paper with 1 mm squares to give a visual representation of 1,000,000 in a 1,000 by 1,000 grid. (She got a kick out of making the class guess what she had a million of in her backpack.) The diagram also included a 10 x 10 square indicating 100, and a 10 x 100 rectangle indicating 1,000. The teacher loved it and kept it for future classes. Other outside projects included a magnet experiment, a large drawing and brief biography of Marie Curie, and a handmade mini electrical generator (which might have produced all of one watt). On entering second grade, Rosie's

weakest area was writing because of physical limitations; however, it ended up being her area of greatest growth.

She was given the school's standard placement tests and screening tools for gifted programming at the end of second grade and she earned scores that placed her above the 90th percentile on the Stanford Achievement Test for reading and math. She scored in the 90+ percentile on the Stanford 9 for reading and math among third graders and on the Otis-Lennon School Ability Test (OLSAT) she scored in the 99.9th percentiles on both the verbal and nonverbal scales.

Because Rosie had forgotten some of her math facts over the prior summer, math and reading became daily activities over the next summer. She completed the HeyMath! website through the fifth-grade unit, and when given *DWP: Grade 6*, she worked (by her own choice) through the first 3 weeks of problems in one sitting. For reading, she conquered *The Lord of the Rings* and many other books. Additionally for language arts, she read a book on grammar entitled *Woe Is I*.

Within the first couple of weeks of third grade, an IEP meeting was convened. School personnel suggested starting Rosie's math by trying to fill in third-grade "holes." Her parents looked at each other and literally burst out laughing. They reiterated the points they had presented in an e-mail in advance of the meeting (that Rosie had completed a fifth-grade online course and sixth-grade word problems) as well as pointing out that the district's own testing in the prior year had placed her in high fourth-grade performance. An agreement was reached to use math contracts with Rosie in a fashion akin to the DT>PI model and to attempt to complete both the fourth- and fifth-grade curricula. There were two major shortcomings. At first, any errors she made in the pretests were simply added to Rosie's contract without further probing into whether she actually understood the concepts and had simply made computational errors. This lengthened the duration of fourth-grade material, but was corrected with the fifth-grade work. Unfortunately, throughout the year, Rosie worked by herself, or in a small group of children (working on different contracts), who received instruction only when they did not understand the material and needed to ask a question. No lessons explaining the material were prepared for

her. On nights when Rosie had no homework, her father taught her advanced concepts from prealgebra as well as problem solving.

Other school accommodations included placement in the highest guided reading group, spelling extensions from Merriam-Webster's Spell It! site (<http://www.myspellit.com>; which also serves as the Scripps National Spelling Bee study site), and quarterly extension projects alternating between social studies and science. One social studies project involved creating a small travel guide to four landmarks in Washington, DC. A science extension entailed creating a crude yet functional wooden xylophone and PVC flute. Rosie also took part in a gifted pull-out program that covered subjects like algebraic concepts using blocks and dominoes, creating and testing structures, solving word puzzles, and studying a unit on a pioneer schoolhouse. One of the most helpful accommodations was a weekly 5-minute phone call from the teacher to Rosie's parents; it kept communication lines open and reduced anxiety on the parents' side.

In February of grade 3, Rosie took the EXPLORE test. When Rosie got home after taking the test, she told her parents she thought she did well; she was able to answer 5/6 of the math questions before running out of time. Her EXPLORE Math score was 17, which is at the 75th percentile when compared to eighth graders, students who were 6 to 7 years her senior. That score also placed her at the 99th percentile when compared to other gifted third graders taking the test, the vast majority of whom had not skipped two grades as she had. Her other scores were English 14, Reading 15, and Science 14, all excellent scores for such a young student. Somewhat amusingly, she left the test during the science section in order to use the restroom (obviously not giving the testing the same weight as her parents). Her parents forwarded the scores to her school and as of this writing are negotiating next year's IEP.

For fourth grade, Rosie's parents are considering several options regarding math. Their highest priority for fourth grade is to move from a system in which Rosie asks questions to initiate instruction to a system in which she receives regular instruction with the curriculum. In other words, they are looking for a planned curriculum for Rosie. A related priority for the next year is for Rosie to be instructed with

one or more peers. School personnel have promised to administer the Orleans-Hanna Algebra Prognosis Test, usually given in her district to fifth graders to see who is ready for prealgebra or possibly algebra in middle school (grades 6–8). Based on the results, Rosie could take sixth-grade math or higher at the middle school, which raises issues in scheduling, transportation, and social adjustment. Alternately, she could be instructed two times per week by a mentor (again, preferably with another student) and work independently the other three days. This could involve the sixth grade or higher curriculum or the M<sup>3</sup> gifted units from the University of Connecticut and/or gifted math units developed by the Center for Gifted Education at The College of William and Mary. The parents are also considering a year’s worth of problem solving from the Olympiad books and participating in a math team coached by her father. Secondary options include online instruction via ALEKS, independent study with *Teaching Textbooks*, or some sort of cyber-school hybrid. Rosie has expressed a strong desire to remain with her friends and not be whole-grade accelerated again.

The story so far takes us to the spring semester of third grade. Her parents fully realize that they will need to continue to advocate for their daughter and attended an end-of-year conference with the following goals, all of which were met:

- Help school personnel recognize the extraordinary level of Rosie’s mathematical talent.
- Discuss the need to plan for a steady diet of appropriately challenging mathematics throughout Rosie’s school years.
- Recognize the importance of allowing Rosie to take tests such as the Orleans-Hanna or Iowa Algebra Aptitude Test in order to answer the question, “Is she ready for algebra?”
- Advocate that she be allowed to advance to a prealgebra class that is appropriately paced and advanced to algebra when she is ready, even though she’ll be quite a few years younger than many “peers” in the mathematics classroom.
- Help school personnel recognize that, for Rosie, there are options that are far preferable to studying math alone in the back of the classroom. These options require putting together an appropriately planned mathematics program for Rosie that

might include placement in a regular class with students several years older, working with a mentor, or participating in online math classes—or a combination of all three options.

- Address with Rosie’s educators the common phenomenon of gifted students’ superior understanding of mathematics concepts, relative to acquisition of basic computation skills.

In a conversation with the district’s Director of Student Services, there was broad agreement that although Rosie is accelerated by 2 years, she interacts well with other students in her class. To eliminate any concern about possible gaps in her mathematics background, she is scheduled to take the cumulative fifth- and sixth-grade math tests. Also, Rosie’s parents assured her educators that they need not be concerned about socialization in math because she has many social experiences outside of math. All-in-all, this is a wonderful example of how good communication between parents and educators, coupled with both parties’ willingness to be flexible, can address the needs of all parties. Most importantly, the student is appropriately challenged and educators have input into the decision-making process.

Rosie’s dad had one final comment about his advocacy for his daughter’s education. He experienced some disdain from other parents because, as he put it, he “actively attempted to challenge her in math.” But, he put it all into perspective with this statement: “Some parents will drive their children one hour each way to take part in ‘travel sports’—and they think I’m a maniac for encouraging my kid to do 15 minutes of math every day and to read for a half an hour. You make the call.” He summarizes his daughter’s progress relative to parental effort with these words: “My original goal was to have Rosie reading before kindergarten . . . oops!”

### **Christopher: “He Wants to Learn Math as Much as He Wants Air”**

Christopher’s parents knew from a young age that he was bright. By the age of 1, he had memorized books and knew the numbers 1–10, and

## Billy: “If He Does Third-Grade Math in Second Grade, What Will He Do in Third Grade?”

According to his mother, Billy had an “unquenchable” thirst for learning and he constantly asked for something harder and more challenging. He was an “extremely intense and driven child.” At the age of 2, Billy became utterly fascinated with numbers. He counted everything, everywhere. He enjoyed doing dot-to-dot books because he was able to follow the numerical order of the numbers. For additional challenge, he began creating his own dot-to-dot puzzles.

Billy also showed an interest in letters and reading at a very young age. For example, at 18 months, he recognized all of the letters of the alphabet. He loved playing with his ABC puzzle. After removing all of the letters and mixing them up in a pile, he would complete the puzzle in correct order with no hesitation. Throughout the day, he searched for letters, pointing to and announcing letters wherever he happened to be (e.g., in the store, in the car). Because of his enjoyment of letters, he loved watching *Wheel of Fortune* from his playpen.

Billy taught himself to tell time and to calculate minutes on a traditional clock at the age of 3. One evening at dinner—out of nowhere—he looked up at the clock and told his parents it was 5:22 and, in 38 minutes, it would be 6:00. That was the first time they realized he could tell time and that he was adding and subtracting. After that day, he began to add and subtract minutes on a daily basis. He became obsessed with calculations. He would calculate anything he could. He also became obsessed with television game shows. He would rush home from preschool so that he could watch *The Price Is Right*. At church, he would take all of the numbers of the songs from the songbook and add them together.

One day, Billy’s mother found him on his bed surrounded by supermarket ads. He was adding up all of the prices with a calculator. One of his favorite activities was going grocery shopping. By the age of 4, he could deduct coupons from the grocery bill total and calculate the change his mother should receive before the cashier did. One incident in particular stood out in his mother’s mind. Her total was \$207.60. She had \$175 in cash, and was planning to write a check for the balance.

Before the cashier could calculate the difference on the cash register, Billy said the check needed to be for \$32.60. The cashier was amazed.

At age 4, Billy knew the correct locations of all 50 states on a United States puzzle. He also knew all of the capitals. He played Yahtzee daily. It was one of his favorite games because of the numbers. He also liked to calculate all of the players' scores in his head. His parents realized he could read when, one night during his bath, he read the back of a shampoo bottle. From then on, he read everything he could and phonetically sounded out what he could not. He began reading first-grade level books at home.

When Billy was 5, he entered kindergarten at a local parochial school. It quickly became evident that he was well beyond his peers and the course material that was offered. His parents were frustrated by his first report card. It showed "satisfactory" performance. It did not show any advanced knowledge or abilities beyond those of an average kindergarten student. The report card stated that he could recognize numbers 1–12, when, in fact, he could recognize numbers in the thousands. He was adding, subtracting, and doing simple multiplication and division problems at home. His report card also stated that he was reading at Level 1, which was the expectation for the average student. Billy's mother took two mathematics workbooks with her to the first parent-teacher conference to show what Billy was working on at home. She attempted to show those books to the teacher as evidence of his ability, asking if the teacher could provide some first-grade work. The teacher would not even look through the workbooks, and she said she couldn't make any notes of advanced ability or advanced work on his report card other than what was expected or "satisfactory." She said she would not give him any extra work and that maybe they should look into sending him to a public school. When Billy's mother met with the school principal, she received the same responses. Like his teacher, the principal wouldn't even look at the workbooks, and it seemed as though she didn't believe that Billy was exceptional.

Standardized testing that spring showed that Billy was performing in the 99th percentile for his grade (Cognitive Abilities Test: Verbal, 85th percentile; Quantitative, 99th percentile; Nonverbal, 99th percentile; and Composite, 99th percentile). Billy's parents became

aware that the public school district had a gifted program in which Billy could participate. He was tested using the Stanford-Binet and earned a Composite standard score of 148 (in the Very Superior or Gifted range), a Verbal Reasoning Standard Score of 127, an Abstract/Visual Reasoning score of 143, and a Quantitative Reasoning standard score of 163. On the Wechsler Individual Achievement Test, Billy's Basic Reading standard score was 149, and his Math Reasoning standard score was 160.

Clearly qualified, he started the district's gifted program in the fall of first grade. He continued to attend his parochial school and was pulled out for 30 minutes twice a week to attend the public school gifted program. Billy's regular first-grade class was filled with worksheet after worksheet, and he was happy to participate in the gifted program because he did a lot of above-level reading (third- and fourth-grade level), math work, and projects.

At home, Billy worked on third-grade math workbooks on his own initiative. He enjoyed a subscription to a magazine filled with math puzzles and logical thinking activities. He especially enjoyed board games involving money and geography. He continued to enjoy grocery shopping—still deducting coupons and calculating change—but now also figuring out the best buy for mom. On one shopping trip, he instructed her to buy a certain brand of raisins because the “price per ounce” made it a better deal.

Second grade was a turning point for Billy and his family. He sat through regular classes at school. He woke up early most mornings in order to work on his fourth- and fifth-grade math workbooks before getting on the bus for school. He still attended the pull-out gifted classes and was very excited by them. For those classes, he had an excellent teacher who realized his potential and need for challenge. Billy enthusiastically worked on several independent study projects, mostly in math. He also developed an intense interest in the solar system. He attended his first C-MITES weekend workshop at Carnegie Mellon University, which he thoroughly enjoyed.

At the first parent-teacher conference of the year, his parents were presented with Billy's Iowa Tests of Basic Skills scores, which indicated he performed at the 99th percentile in many areas. Billy's mother asked

his teacher about giving him more advanced work. They discussed the possibility of having him sit in on third-grade math classes, but the idea was vetoed by the principal, who said, "If he does third-grade math in second grade, what will he do in third grade?" The only challenge he was given was a sheet of third-grade math homework on a fairly regular basis.

Up to this point, the parochial school had done very little to support Billy's exceptional abilities. Instead of the administration being proud to have a student like Billy in the school, they thought of him as a problem. In turn, his parents were viewed as problems for being his advocates. School personnel didn't know how to handle the situation or what to do with Billy. His parents found this lack of school support and interest quite unexpected. They quickly realized that the role of playing advocate for their son wasn't going to be an easy one.

Several years before, Billy's mother had contacted Dr. Lupkowski-Shoplík at Carnegie Mellon University and had received a lot of information about mathematically talented youth, some of which discussed mentoring. His parents decided that, if Billy could be placed in a mentoring program at his school, it might solve their problems. His mother discussed this idea with the public school gifted teacher, who thought it sounded like a great idea and even volunteered to accompany Billy's mother to the meeting scheduled with the principal and Billy's second-grade teacher.

Although the principal said she previously had not heard of a math-mentor program, she was willing to discuss the idea with the Diocese (the supervisory body for the parochial schools in the area) and gain approval from her supervisors. As a second grader Billy then took the STEP Mathematics Basic Concepts test. His score was at the 46th percentile when compared with the sixth-grade norm group. He scored at the 37th percentile on STEP Computation when compared to sixth graders. Billy also took the second- and third-grade end-of-the-year math tests. The results of these tests showed his mastery of the second- and third-grade material. Clearly, Billy was performing well above grade level in mathematics.

Testing occurred in February, and Diocesan approval was received in early May (a lag of 2 or 3 months between testing and the

administrative decision is not unusual). With only about 6 weeks of school left, Billy was finally able to begin working with his mentor, with whom he met twice a week in lieu of his regularly scheduled math classes. On the remaining 3 days of the week, he worked on mentor-assigned homework and e-mailed his mentor with his progress or to ask questions. He loved it. Billy and his mentor worked on permutations, scientific notation, associative and commutative properties, and probability games.

Billy's mother remembers that he had experienced some social difficulties in school during second grade. He got the distinct impression that it wasn't "cool" to be smart. The other students seemed to resent his intelligence and his enthusiasm for learning. In one math class, another student beat him at flashcards, and the whole class cheered. He was crushed and couldn't understand why kids who used to be his friends were treating him this way.

Although the mentor program was wonderful and Billy and his family were very pleased with the results, it targeted only mathematics for Billy. They thought he needed acceleration and challenge in other areas, as well. They decided to look into other schools. Billy was excited about the idea, saying that it would be nice to attend a school that would be challenging and also where there would be more "kids like me." They found a local private school that seemed to be a good fit, and he began attending that school in third grade.

Billy's mother said,

We are very content with our decision. As parents, we were searching for a school that would provide challenge without being overwhelming and where [Billy] could be the enthusiastic, inquisitive student he needs to be. From the time he was a toddler, he has had an unquenchable thirst for knowledge. We believe his new school provides him the atmosphere in which he can thrive. Our beliefs were confirmed when, at our first parent-teacher conference at the new school, his teacher said that [Billy] was at the top of his class and was "made" for the school. Because of his personality and his intense desire to learn, the transition to his new school has been a very smooth

one, as indicated by all of his teachers. He is finally happy and content. He is with other children like himself where it is acceptable to be smart, and he is able to accelerate within the class. For example, in reading, his teacher is recommending he read fifth-grade-level books for his monthly book reports. In math, he is in a small group that is working on the same subject matter as the rest of the class, but at a higher level and a quicker pace. Billy has his own math folder, as do other students, which is filled with math activities geared toward his ability: critical thinking, logical thinking, algebra, and so forth. There are several centers in the classroom, and students are able to choose a center at which to work. The key here is that he is not being restricted by anyone, and an entire class is not being rushed ahead because of him. The administration and teaching staff are supportive, and they are thrilled to have Billy and students like him attend their school. What made him so difficult at his former school has impressed his new school.

Our parting with the previous school was amicable. They understood our position and said they couldn't structure their curriculum to accommodate students like Billy. They had to target the average student. We believed our son had so much potential and that it would be an injustice to let him remain in a school where it wasn't possible for him to develop to his full potential.

Billy took the EXPLORE test in third grade and earned the following scores and percentile ranks, which are comparisons to eighth graders: English 17 (73rd percentile) Math 15 (62nd percentile) Reading 12 (44th percentile), Science 17 (76th percentile), and Composite 15 (61st percentile). His performance on the EXPLORE test was exceptional. Specifically in mathematics, third grader Billy had outperformed the average eighth grader, who had 5 more years of experience in mathematics than he did. Once again, Billy's above-level test scores indicated his tremendous abilities and potential in mathematics and his need for differentiated programming.

Personnel at Billy's new school went to great lengths to create a mathematics curriculum that would satisfy both Billy and his parents. Billy's parents met with the school headmaster, the fourth-grade teachers, the upper school math teacher, and other parents to discuss ideas for the upcoming years with regard to Billy's mathematics curriculum. For fourth grade, Billy and five other students were separated into an accelerated mathematics group and were taught fifth-grade math. It worked well.

For fifth grade, after careful consideration, school personnel moved this group of six fifth-grade students directly into the sixth-grade accelerated math class. It was also decided that this acceleration plan would follow this group of students through the eighth grade. Again, this plan worked out extremely well. The sixth graders accepted the group of fifth graders immediately, both socially and academically. Billy was at the top of his class. He also participated and placed in several math contests at the sixth-grade level.

Presently, Billy is in the sixth grade, and he attends the seventh-grade accelerated prealgebra classes. He is at the "right" grade level for his age, although he is accelerated in math by one year. In addition to his prealgebra class five times a week, he attends a math computer class weekly. He is at the top of the class and has participated in several math contests at a seventh-grade level (e.g., Continental Math League, American Math Competition). He has earned straight A+'s in math since he started attending the private school in third grade.

Next year, when Billy is in seventh grade, he will attend eighth-grade Algebra I and plans to participate in several math contests offered at his school. He will also be eligible to try out for the school's MathCounts team. He expects to take Algebra II in eighth grade.

Billy's parents are very pleased with his progress. They believe that their decision to move him to a different school was critical. Billy is now in an environment where his talents are valued and nurtured. His mother commented,

It is not easy to parent a gifted child. Over the years, we have had to hurdle many educational roadblocks because of the guidelines established by standard curricula. Most of the time,

parents are the only advocates a gifted child has. We also learned that standing up for your gifted child is not always the “popular” thing to do. In fact, when we made the decision to move our son from the parochial school to the private school, certain parents [from the parochial school] started to act differently toward us. One comment was, “He can’t be that smart. Who do they think they are?” Why should we feel the need to apologize for our son’s exceptional abilities? But, that is exactly how many people have made us feel. . . . These thoughts of mine are important, I think, to other parents who are just beginning their journey into the world of educating the gifted child. I know that, when I first read *Jane and Johnny Love Math*, [*Developing Math Talent* is a follow-up to *Jane and Johnny*] it was comforting to realize that our family was not alone.

We are pleased that Billy and his parents are content with the decisions they have made and with the curricula that have been made available to Billy at the private school. We think it is interesting, however, that the private school made such an issue out of accelerating their mathematically talented students by one year. Clearly, the students were ready for that type of acceleration, and Billy might have benefited from even more radical acceleration in mathematics. In relation to that, we wonder why there was any question about what the students should do after completing fifth-grade math. What other choice should they have except to go on to sixth-grade math? Should they have repeated fifth-grade math? This lack of logical thinking doesn’t surprise us because we hear these stories frequently, but we are always disappointed by it.

### **Arthur: “Is He Being Challenged Enough?”**

Mrs. A. first contacted us when Arthur was a first grader in a suburban school district. She explained that he was in the school’s gifted program and had demonstrated that he could do fourth-grade work. Arthur had shown high ability in mathematics since he was quite

## **Elizabeth: A Scientifically/Mathematically Gifted Female With Parents Who Did Not Want Her to Be Different**

We became aware of Elizabeth because of a phone call from her mother, which occurred after many years of discussions (but little action) with school personnel. In those discussions, the most specific issue concerned whole-grade acceleration and its pros and cons. At that time, Elizabeth was in fourth grade and her mother was concerned about the lack of academic challenge in her daughter's education. Elizabeth's mother was worried that an unchallenging curriculum would have a negative impact on Elizabeth's self-esteem, possibly leading to boredom and eventually to acting-out behaviors. At the same time, Elizabeth's mother expressed concern that any recommended placement options not make her daughter "different from her classmates."

Our initial response to her request for our consultation was to highlight the need for specific, objective information prior to making any educational programming decisions, including acceleration. We also suggested that it would be best for the school to conduct any assessments (with our consultation) so that they would feel more ownership of the results, as well as the recommendations. We thought it was important to have collaboration among the school, the parents, and us. Fortunately for Elizabeth, the collaborative process was possible because of the very capable gifted education teacher who worked with her in the school's enrichment-focused pull-out program.

Elizabeth was one of the most able students in her school's pull-out program, which consisted of an hour of enrichment per week. However, her teacher for this enrichment program was the first to recognize that this level of programming was not providing Elizabeth with the systematic, high-level curriculum she needed in mathematics and science.

The first step in identifying Elizabeth's educational needs was to see how she compared to others in her grade. Elizabeth had consistently scored at the 99th percentile on the Iowa Tests of Basic Skills (ITBS); a careful examination of the item profile for her ITBS results indicated that she never missed an item. It was obvious that she was talented—

## Table 9.1

Elizabeth's EXPLORE Math and Science Scores in Fifth Grade

EXPLORE test	Standard score	Talent search percentile	Eighth grade percentile
Math	13	48	48
Science Reasoning	21	93	93

achieving at a level that was higher than others in her grade—but the grade-level assessment could not reveal the level for which curriculum/instruction was most appropriate for her. The only “given” was the fact that Elizabeth and her parents were convinced that she needed a challenging mathematics curriculum. We decided to administer the quantitative sections from an above-level test. On that test, while in fourth grade, Elizabeth scored at the 79th percentile when compared to sixth graders.

We then used the Sequential Tests of Educational Progress (STEP) tests that were normed for grades 5–8, and compared Elizabeth’s scores to spring-semester sixth graders. Elizabeth earned a score at the 87th percentile on the STEP Basic Concepts. On the same level of the STEP Computation test, Elizabeth earned a score at the 25th percentile—a score that was considerably lower than her Basic Concepts score. This did not surprise us because we had already anticipated that her computation skills, like those of many mathematically gifted students, lagged behind her conceptualization of mathematics (Rotigel, 2000). An interesting observation concerned the fact that Elizabeth was able to earn a high Computation score when she was allowed to complete the items in an untimed setting. Nevertheless, it was apparent that she needed to have a firmer foundation in computation. From the results of the diagnostic testing, we recommended that Elizabeth begin prealgebra as soon as possible. A late-spring meeting in Elizabeth’s fourth-grade year that included teachers, parents, the principal, and us was set to identify instructional goals for systematically working through the prealgebra curriculum during fifth and sixth grade.

During these discussions, we addressed the parents' concerns about Elizabeth being "different" from her classmates. Interestingly enough, this was not and still is not a concern for Elizabeth. We also discussed the parents' concerns that their daughter might be stressed by doing advanced work. We shared with them our perspective on the differences between "stress" and "stretch." Because the parents felt the need for further discussions regarding their daughter's giftedness and its impact on her and their family, they were referred to a family counselor who had specific training in working with issues unique to families of gifted students.

Our impressions of Elizabeth were that she seemed like many academically able girls: self-effacing with respect to her abilities and simultaneously anxious to please her teachers. Fortunately, she had both a classroom teacher and an enrichment program teacher who very much wanted to see Elizabeth appropriately challenged and were willing to work together to differentiate the curriculum to do so.

These two teachers made a 1-year curriculum last for 2 years by "stretching" the prealgebra curriculum through fifth and sixth grade. Also, when Elizabeth was in fifth grade, she participated in the Elementary Student Talent Search. Her scores on EXPLORE were superior (see Table 9.1) and demonstrated exceptional scores in both math and science reasoning. Because she seemed to be appropriately "stretched" in science through an enrichment project in which she built a camera, less emphasis was placed on providing specific programming in this subject area.

When Elizabeth was in seventh grade, she was allowed to take algebra, which at that time was a rare occurrence for her district, because mathematically able students were only accelerated into algebra in grade 8. Unfortunately, she didn't have any math as an option in her school when she was in eighth grade, and she did not have the transportation to get to the high school for additional math classes. Because the district was not willing to provide programming for her, she played with a computer in the back of the mathematics classroom during her eighth-grade year. At that time, there was still not much available for summer programs in her area, so she was unable to take advantage of that sort of opportunity.

However, once she entered high school, Elizabeth took full advantage of the opportunities. She took as many science classes as were available, often “doubling up.” She completed the college-entrance requirements fairly quickly and participated in the postsecondary enrollment option available in many districts. This opportunity permitted her to do research with a physics professor throughout her junior and senior years. More importantly, working with a college professor while still in high school gave her added confidence. She also got back on track in mathematics and completed calculus as a senior. When it came time to consider colleges, she applied and was accepted at prestigious technical institutes on both coasts, but chose the institution that fully covered her tuition (and is located on the warmer of the two coasts). Her declared major was physics. Later, she earned a master’s degree in earth science and as a young adult has found employment with an environmental consulting firm.

In the first conversation with Elizabeth’s mother, the concern about her daughter being “different” as a result of a program intervention was a major point of discussion. We clearly remember advising the mother that, with respect to a need for a very challenging curriculum, Elizabeth was different from her classmates and that our consultation would provide objective information that could be used to develop such a curriculum.

If we knew then what we know today about mathematically (and scientifically) talented students, we would have done things slightly differently in three areas. First, we would have addressed Elizabeth’s talent in science, as well as in math. Additionally, we would have strongly insisted on long-term planning in both of her talent areas. This would have avoided the lost year in which Elizabeth sat at the back of the classroom and worked on the computer. Finally, we would have tried to “normalize” Elizabeth’s talents. That is, we would have encouraged the mother’s efforts at advocacy for her daughter and emphasized that her daughter is normal with some unique academic needs. Placing Elizabeth with other gifted students (such as in special classes, summer programs, and academic competitions) would also have given Elizabeth and her parents the chance to get to know other

talented youngsters and helped Elizabeth to realize there are other students who have the same academic interests and needs as she does.

## **Zach: The Preschool to Graduate School Journey: What Parents and Teachers Need to Know**

Zach is a doctoral student in physics and likely on the path to an academic career where he will continue doing what he does as part of his doctoral program: reading papers, consulting with colleagues, working calculations and thinking about physical models, drafting papers and presentations, and attending presentations. Zach and his parents are, understandably, the heroes of his story, but so too are his elementary gifted education teacher and other educators and administrators who made decisions or used teaching strategies that positively influenced Zach's academic and personal life. His journey from preschool to graduate school serves as an inspiring testimony for the necessity of parents and educators to work together to ensure a differentiated and challenging curriculum.

Zach is the firstborn of three boys, all gifted. His precocity with letters and numbers began when he was a toddler and was apparent to his father, a university professor and administrator, and his mother, a physician. Zach has always been an avid reader; when in elementary school he especially enjoyed novels and comics and thought he would grow up to become a novelist or cartoonist.

Information from his parents coupled with his outstanding performance on standardized tests validated Zach's self-awareness of his academic giftedness. Erringly similar to observations made by the expert mathematicians in Bloom's (1985; see previous chapter) classic study, Zach's comments about elementary subjects—especially math—are an important part of his story:

Elementary math is learning long division, in science one learns the parts of a flower or an ant, and in social studies one memorizes the countries of South America. In reading and writing however we had much more opportunity to explore

talented youngsters and helped Elizabeth to realize there are other students who have the same academic interests and needs as she does.

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Elementary math is learning long division, in science one learns the parts of a flower or an ant, and in social studies one memorizes the countries of South America. In reading and writing however we had much more opportunity to explore

material that we found interesting, and that was at our level. . . .

Looking back on my elementary years now, I am forced to conclude that I spent a great deal of time learning facts, most of which I cannot now recall. That's not to say that students shouldn't learn factual information, but I don't think that that should be a goal in and of itself (which I think is how some teachers genuinely regarded it). It seems to me that the most important things my teachers did were to motivate, socialize, and to teach certain skills.

Outside of school, reading and the game of chess would prove to be important leisure activities for Zach. In school, Zach recalls that his fifth- and sixth-grade homeroom teacher provided substantial personal feedback on his reading journal and also engaged with him in intellectual discussions. When he was in fifth grade, his gifted education teacher became his prealgebra teacher and in sixth grade, he traveled to the middle school building to take algebra with seventh graders. Contrary to the implicit theories of some educators, which either assume that elementary students do not want to be in classes with middle school students or that middle school students will make fun of the younger student, Zach had a very positive experience with subject acceleration.

In sixth grade I traveled by van to [the junior high school] to take Algebra with Mr. D. I was quite proud of it, and I believe I found math increasingly interesting, probably because the subject matter in math simply was becoming more interesting.

I didn't like taking the van, which meant getting up earlier and missing other activities at [the elementary school]. Other than that, I don't recall finding the homework or other aspects of junior high education particularly burdensome. *I believe now, and believed then, that it was easily worth the extra effort [emphasis added].* It was inconceivable to me that I would not have a math class, and I was told that taking the van to [the junior high school] was the best way to do it.

Zach's parents were continuous advocates for their son and, as Zach was finishing elementary school, his parents and educators, led by his elementary gifted and talented teacher, advocated for whole-grade acceleration in addition to the single-subject acceleration in math already in place. In fact, Zach's recollection of the process was that he did not actively pursue the option or even had input on the final decision, but he was kept informed during the process. Zach's insightful comment about whole-grade acceleration, as it pertained to him, captures the rationale generally expressed by gifted education professionals:

I also believe my parents said that the motivation was not so much that [junior high school] would be especially challenging but that the acceleration would get me closer to the high school honors classes. I agreed with their reasoning, and certainly preferred more challenges to being bored in school. I recall feeling that I was being pressured to do something hard, but that I agreed with the reasoning.

Zach skipped the first year of a 2-year junior high school program and went from grade 6 (elementary school) to grade 8 (junior high school). This approach is supported by the *Iowa Acceleration Scale* (Assouline et al., 2009) process, which stresses that the final year in a building before going to the next level, known as the transition year, is critical to the success of an accelerated student. Zach remembers feelings of social isolation at the beginning, which he attributed both to the newness of the setting and the fact that he was younger than most of the students in that grade.

The quasi-gifted class [with a focus on leadership and career exploration] was unmemorable academically, but did introduce me to a number of people with whom I would be good friends for years to come. My favorite class was Geometry. I spent the whole class working on math problems with other students. The teacher helped us when we got stuck and occasionally told us interesting stories.

When regular educators or school administrators are unsupportive of academic acceleration, the most common reason is rooted in the concern that children who are accelerated will not adjust well socially to the new class (Colangelo et al., 2004a). The research countering this argument is robust (see, for example, Colangelo et al., 2004a). Zach eloquently supports the research with his personal experiences:

Most socializing in school happens without direct intervention by teachers. In class, I think it's important for students to work in groups. I think it's very important for a gifted student to not always be the smartest person in the room (or group). Placing a gifted student with other gifted students and/or older students seems to me beneficial for this reason. In high school I often had class with students two or three years older than me [*sic*], and generally preferred their company to that of my age-mates (who were by then a grade behind me). *On the whole I think that acceleration eventually helped me socially by allowing me to meet similar students. I think that an educational program can't be considered particularly successful if it has a negative impact on the child's social development* [emphasis added].

With respect to whole-grade acceleration, it is impossible to say with certainty that such a decision, at that moment in time, is the right one for any particular individual. That's why it is so important to hear from individuals regarding the long-term impact of acceleration. Extensive research (see Lubinski, 2004) on groups of accelerated individuals reveals that the long-term impact is positive, and the only "regret" is that individuals didn't accelerate sooner or experience more accelerative opportunities. Zach gives voice to the group data:

I definitely think that skipping seventh grade was the right decision. Accelerating further would probably have been possible, but not [necessarily] beneficial. Even knowing where I would end up (and if I had to do it over again, I would end up right where I am now), I don't think there is any reason to narrow my academic focus earlier than I did. There are cer-

tain classes I could see myself skipping, and I do wish there had been some way to take another math course in the spring semester of 12th grade. Ideally, I would have loved honors classes stretching all the way into junior high and perhaps elementary school, though I understand there are downsides to such “tracking.” It would be bad if I only interacted with gifted students, but it’s not clear to me what the downside would be if my academic interactions were more restricted to other gifted students, relegating interaction with a broader spectrum of my age-mates to nonacademic activity.

Because Zach took geometry as an eighth grader, he was on a trajectory for subject acceleration that would place him in calculus his junior year in high school. Zach’s comments about this class and his teacher once again support our points about the need to have students in challenging classes that are taught by teachers who themselves are lifelong learners (see Chapter 8).

Calculus was by far the most enjoyable of my many math classes, before or since. It answered so many questions that I had (tangent lines, areas, maximization), and gave the impression of being extremely useful. The teacher was an enthusiastic doctoral candidate [in math education] with the improbable name of [Demi Pi]. Perhaps it was because he himself was focused so hard on math and so enthusiastic about it (as opposed to teachers with only mild interest in the subject matter) that the class was so interesting.

My senior year I took Honors Linear Algebra at [the university] through the school district’s Post-Secondary Enrollment Option. I think many more students than actually utilize this program would benefit from it. Linear algebra was more abstract than I was used to, and also much more difficult. I had had homework before, but working for 10 or so hours a week and still not being able to solve many of the problems was shocking. Later, the freshman linear algebra course at [my undergraduate university] was a breeze. I felt

good about taking the course, although I might have been bitter if the generous curve hadn't saved my class grade in the end.

Unfortunately, there was no way to schedule an appropriate math class around my high school schedule in the second semester. I ended up taking a programming class in C++, which I haven't used since then. The material was similarly difficult, but not interesting or useful. I was also somewhat bitter about my first B+, which I attributed to a rounding error on the part of the [university] professor (he acknowledged it, but refused to correct the record).

Zach's recounting of his high school years is more detailed than space permits. In brief, he indicated that throughout high school he remained an avid reader but also spent a lot of time on chess tournaments and math club. In high school, he became very competitive in adult chess tournaments—he was the state champion. However, he did not continue with competitive chess during college for two main reasons: (a) his undergraduate courses were more challenging than high school, and (b) Ultimate Frisbee, which had been suggested by his mother, became his main leisure activity. In reflecting on the benefits of participation in Ultimate Frisbee during his undergraduate years, Zach commented that he enjoyed the competition, exercise, and social aspects of the game, even though he acknowledges that his team was more often on the losing side than the winning side. Also, the Ultimate Frisbee teams were slightly “less nerdy” than the chess competitors. He continues to play Ultimate Frisbee.

High school math club was a different story. Zach describes his high school's math club as a “serious force,” with opportunities for regional math competitions where he made a number of friends and saw that others were sometimes equally talented or even more so. Also, the math club provided an important opportunity for leadership. In reflecting upon the role of competition in his cognitive and social development, he commented that perhaps he was “too competitive,” and suggests that teachers should not use competition to motivate their gifted students, but to challenge them to do their best, “which is often

better than they [the students] think they can do rather than to focus on doing better than their classmates.”

By his senior year in high school, Zach recognized that math and science were areas in which he was more able and more interested, relative to language arts. Although he applied to both general liberal arts colleges and technical universities, his visit to a private technical university sealed the decision to attend that school. He started out with chemistry as his major and—almost by chance—took a class in physics that he might have passed out of. Zach credits the excellent physics professor with making the experience much more enjoyable than that of his chemistry classes. He also declared a second major in economics, but realized that it was an area that would not be pursued in a graduate program.

My two majors used math to describe broad sections of the physical and human worlds, which is what appealed to me about them. From that, we might reasonably say that it was physics’ methodology, rather than the particular physical systems involved, that attracted me. I like solving problems and explaining things, and my best tool is math.

Zach’s teachers and parents have provided nearly constant support, which has resulted in the maximization of his academic experiences. Zach clearly articulates the distinction in roles between parents and educators:

Schools have a responsibility for the education of the student body as a whole, but it’s up to parents to hold schools accountable for educating an individual child, particularly one [who] would benefit from special programming.

I would say that it is important for a child to be challenged in school, if for no other reason than that they [*sic*] need to become accustomed to facing challenges. In that sense a gifted student enduring regular programming not only misses the opportunity to acquire advanced skills, but fails to receive the normal training that an average student experiences. Further,

many careers which would challenge a gifted student (I think in particular of my own) expect the skills that can only be developed through advanced coursework. That is to say, the benefits of a personalized curriculum are helpful, if not essential, to success in a gifted student's likely life goals.

These statements provide solid support for parents and educators who advocate for program change to ensure that a curriculum is appropriately challenging. If a parent doesn't advocate for the child, who will? But advocacy cannot take place in a vacuum—the application of a differentiated, challenging curriculum must come from the educational system and is maximized when in-school and out-of-school experiences are combined.

## Lessons Learned

As a result of working with Rosie, Christopher, Billy, Arthur, Elizabeth, Zach, and many other students, we have learned numerous lessons.

1. Parents know their kids. Their advocacy is a critical aspect of their child's academic development.

The parents of the students in our case studies noticed their young child's unusual facility with numbers. Christopher was 3 years old when his parents noted his precocious ability with numbers. As soon as he began expressing his observations about numbers, they began to take note, and their observations proved extremely useful in alerting educators to Christopher's abilities.

2. Mathematical ability can be recognized at a young age. Some of the early indicators of mathematical ability include:
  - Telling time early and showing an early interest in numbers (e.g., counting objects; Waxman et al., 1996a).

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  - Telling time early and showing an early interest in numbers (e.g., counting objects; Waxman et al., 1996a).

- An early sustained interest in manipulating numbers (e.g., 3-year-old Matthew spontaneously added the numbers he saw on a truck,  $369 + 369$ , and shouted out, “738!”).
- An ability to construct their own understanding of mathematics without direct instruction (e.g., Peter demonstrated an intuitive understanding of multiplication at the age of 2. He was counting out three raisins for each gingerbread man he and his mother were making. When he was told they would make four more cookies, he said, “Then we need 12 more raisins!”).

These early indicators of mathematical ability are first observed typically by the parents, which again illustrates our earlier point that it is critical to listen to parents’ stories about their children. However, the precocity of some mathematically talented students is not necessarily realized before the student starts school. At the time, Zach’s parents did not think the things he did at a young age were especially remarkable because he was their firstborn and they didn’t have any other young children with whom he could be compared.

3. Because school personnel often take a “wait and see” attitude or just say “no,” parents need to be persistent in their advocacy efforts.

Over and over again, we hear stories about parents being dismissed by school personnel or told to “give it more time.” Some of our favorite quotes include: “All kids need enrichment and that enrichment will satisfy his or her need.” “We don’t do acceleration here.” “We accelerate only 1 year.” “No student is ready for algebra before age 13 or 14.” These statements have been made by real school personnel, and these ideas represent educational myths that we have attempted to dispel throughout this book.

We’ve also heard interesting stalling tactics, such as, “Let’s put Anthony in the regular classroom this fall and see how it goes. If he has trouble, we’ll test him in November. Then, it’ll be Christmas vacation, and we shouldn’t start any special programs. Let’s plan to

start something in January.” This is why it is important to plan in the spring for the following school year. Then, the principal and guidance counselor (and whoever else is involved with scheduling) can take into account a child’s special needs when doing the scheduling. Christopher’s parents did an excellent job using this strategy.

We have found that parents and gifted education teachers are usually the driving force behind discovering mathematically talented students and generating appropriate programming for them. Both parents and gifted educators need to be persistent at creating an appropriately challenging program. For Zach, his gifted education teacher was also his fifth-grade prealgebra teacher and that made all of the difference to his program. The circumstances with Zach evolved into a positive learning environment with his best interests in mind. Oftentimes teachers have less-than-adequate support from their administrator or from colleagues. We have sometimes experienced the situation where the teacher is willing to give a student more advanced material, but the principal will not support it. This is a difficult situation that can require the negotiating skills of a professional diplomat! It is always helpful to have an advocate within the system, but it is not appropriate to put that advocate at risk in his or her job situation.

4. School-based assessments are typically grade level in nature, and many educators are not aware of the need or procedure involved in above-level testing.

Billy’s teachers used the standards for the primary grades as indicators of his achievement (“can recognize numbers 1–12”), but didn’t attempt to measure what he could do beyond these minimum expectations or what he was ready to learn next. The information we presented in Chapters 3, 4, and 5 should be helpful in determining what level of test to give to a student and how to interpret the resulting scores. As we have stated repeatedly in this book, the goal is to determine what the student already knows and then move on from there. Rather than using grade-level tests that tell us what we already know (the student is performing very well compared to age-level peers), we offer students the opportunity to take a much more challenging test and

demonstrate their exceptional abilities. Although the approach we advocate is common sense, few educators have had specific training in providing this type of testing and programming for gifted students.

5. All assessment (testing) needs to be driven by a question.

In our case, the question is, “What is the extent of the child’s mathematical ability and what types of programming modification does he or she need?” In this situation, IQ testing may not be necessary. Although an IQ gives a sense of overall ability and provides guidance regarding the pacing, it does not give the specific information needed to develop a systematic mathematics program. Billy’s testing in kindergarten and first grade pointed out that he was an exceptionally bright child, but it didn’t yield specific recommendations for his math program. The only obvious benefit from his IQ testing was that it helped to identify him for a gifted program, and this resulted in his working with an understanding teacher who became one of his advocates.

6. The Diagnostic Testing→Prescriptive Instruction model is useful for helping mathematically talented youth study mathematics at the appropriate level and pace.

The skills and content students have mastered in mathematics must be assessed carefully to ensure that instruction begins at the proper point, and the DT→PI approach provides the systematic means of doing so.

Both Zach and Elizabeth exemplified some of the issues that mathematically talented kids have to deal with in a district that focuses primarily on enrichment. They were lucky because they had teachers who were willing to go outside of the imposed enrichment curriculum and seek assistance through additional testing (DT→PI) to get the right “dosage” of mathematics. With Elizabeth, it wasn’t perfect because there was no programming in eighth grade. For Zach, possible gaps in his math program were not an issue because the system was already in place when he needed it. (As Zach noted, the exception to this was the second semester of his senior year.)

7. Even extremely talented students need time to develop the cognitive structures that characterize mathematical maturity.

We are strong advocates of acceleration because it is often the only way in which a student can be truly challenged in mathematics. At the same time, we recognize that students (especially very young students) need time to develop intellectually. We are concerned that students who rush ahead before they are ready might learn advanced mathematics by rote without the benefit of deep understanding.

It is important for students to understand the logic underlying elementary mathematics problems before advancing to abstract concepts. We are also concerned that students not have large gaps in their mathematical backgrounds. Thus, while acceleration is appropriate for many mathematically talented students, at the same time it is being planned it is critical to look at how the time saved will be used. We also advocate using the DT→PI model to ensure that students do not have gaps in their mathematics background (see Chapter 4).

8. Mathematically talented students should study mathematics at a steady rate.

If a mentor-based program is implemented, it is not necessary for a student to meet individually with a mentor for several hours each week to advance in mathematics. Two 1-hour sessions with a mentor, with homework assigned between sessions, might be all of the challenge and stimulation a student needs. Students who do accelerate in mathematics also have the luxury of more time for studying enrichment topics in mathematics. This extra time should be put to good use. We encourage students to study enrichment topics that might not be a part of the regular curriculum (see Chapter 7).

Continuous study of mathematics 12 months out of the year is not necessary, and students do not need to race through the standard sequence of mathematics. What happened to Zach should be avoided: He could not take a math class his second semester of his senior year.

Long-term, continuous planning is essential for these students whose needs are very different from the norm.

In a similar vein, students do not need to continue an intensive study of mathematics during the summer. Consider other opportunities, especially participation in summer programs, or activities that allow the child to interact with intellectual peers. These programs and activities might be completely unrelated to mathematics or may be connected. For example, the student may find that an enrichment course in science is not only interesting and fun, but actually enhances the study of math!

9. Choices made when students are in elementary school may affect their high school mathematics program. Educators often fear that “students will run out of math” before they graduate from high school.

If the level and pace of instruction are matched to the abilities of mathematically talented students, by definition they will be accelerated. They may run out of mathematics courses that are offered by their school district before they finish high school, but there is plenty of mathematics to be learned. Students may need to take college courses part-time, work with a mentor, complete distance-learning or online courses, or enter college early in order to have access to an appropriate level of mathematics (see Chapter 6 and the Resources). For example, a young student, like 7-year-old Rosie, is well on her way to radical acceleration in mathematics. Therefore, it is critical to implement long-range planning so that (a) she develops the cognitive maturity to thrive in accelerated coursework and (b) she, her parents, and the educators in the system understand the ways in which her program will need to be adjusted to accommodate her needs.

10. Students may need to look outside of their school system for appropriate programs in mathematics and/or general enrichment opportunities.

These opportunities include summer and weekend programs offered at universities, online programs, mentor-paced programs, and competitions. All of the students included in our case studies took

advantage of programs offered by local universities, zoos, museums, and other educational groups, and their parents were constantly on the lookout for new and interesting opportunities for their children. (The Resources section provides information about many of these opportunities.)

Zach's retrospective on his experiences led to the recognition that such programs provided

challenge [for] them as well as contributing to their social development and emotional maturation. The standard should not be, however, to present the student with the greatest academic challenge they [*sic*] might possibly meet. Too much schoolwork can discourage or unduly stress a student, or crowd out other important parts of life. There are of course resources for students outside of school. I heartily recommend intellectual pursuits such as chess, and private academic programs such as the ones offered by [university-based precollege programs].

11. Concerns about a student's social development shouldn't stop educators from making pedagogical adjustments.

There are a couple of ways to think about the relationship between social-emotional adjustment and academic challenge. In some cases, when students are appropriately challenged, their social-emotional adjustment seems to fall into place. It is also important to help mathematically talented students achieve a balance in their lives while also making sure that they study mathematics at the appropriate level. One of the most effective ways to enhance social development is to ensure that students have time to study other subjects, to play with friends, and to be alone.

Also, most accelerated students still have the opportunity to be with same-age peers in many other situations, such as scouts, religious youth groups, recess, and lunch, even if they are not grouped with their age peers for any of their academic activities. We encourage students to try out different types of activities, including sports and music, not

in an effort to be “well-rounded,” but in an effort to be exposed to different situations and different people.

12. Mathematically talented girls may have special needs.

Research and personal experience has shown that it is important that mathematically talented girls be placed in a challenging, supportive atmosphere so that their talents can be fully realized. As we have mentioned elsewhere, we find many more mathematically talented boys than girls. Many more parents of boys than girls call us seeking our advice. This seems especially true with the youngest students. We have also noticed that parents of girls seem to be more concerned about social issues, stress the fact that they don't want their daughters to be different (as in Elizabeth's case), and are more willing to allow school personnel to talk them out of seeking accelerative options or other special educational programming.

We recommend that parents of mathematically talented girls be especially aware of the social pressures that encourage girls not to demonstrate their exceptional abilities. These students benefit from opportunities to explore mathematics in a supportive atmosphere, work with female role models, and study mathematics with other similarly talented girls. Experiences provided by programs such as those listed in the Resources may be especially important for their mathematical development. (A more detailed discussion about issues affecting mathematically talented females is presented in Reis & Gavin, 1999).

13. Talented students benefit from finding an intellectual peer group.

This is why academic summer programs and weekend programs are so valuable. Talented students can be placed together with others who have similar interests. They enjoy the rewards of being challenged intellectually while also becoming friends with students like themselves. Christopher and Zach both benefited from this type of experience during their regular school day; they were grouped with other mathematically talented students who also happened to be older than they were. The benefits of this type of grouping include the realization

that they are not the only ones who like and are good at math. This experience may be especially important for mathematically talented girls.

Although mathematically talented elementary and middle school students may not have many intellectual peers during the K–12 years, they are usually able to find peers once they attend college. This may also be possible while they are still in high school, as was the case with Zach. All parents worry about their child’s social-emotional development and Zach’s were no exception. Their son is now in graduate school and they have the “luxury” of knowing that their highly gifted child has grown into an individual who is a “good citizen,” as Zach’s dad says. Teachers and other parents may find it useful to know that Zach’s father believes that, if he had it to do over again, he “would have worried less about the social aspect of Zach’s development.”

14. Most parents of talented students we have met are not “pushing” their children.

Most of the parents with whom we work are simply responding to their child’s needs and interests. These parents have not been showing their children flashcards from birth. Billy’s case illustrates this point clearly. He spontaneously demonstrated his capability to tell time, calculate, and read; his parents did not teach him these feats directly. Instead of pushing their children, these parents are being pulled by their children, who consistently demand, “More math!”

## Conclusion

The case studies in this chapter illustrate many of the points we have made throughout this book. Mathematically talented students can be identified at a young age, and they benefit from programming that matches the curriculum and pace of instruction to their specific academic abilities and needs. Talent searches, first pioneered by Julian Stanley in 1972 for seventh graders and now offered by several universities for elementary, middle school, and junior high school

students, provide a systematic method for identifying precocious youth and providing appropriate programs for them. The Diagnostic Testing→Prescriptive Instruction (DT→PI) model, used by the talent searches and by a number of schools, has been effective in helping exceptionally talented students to study the level of mathematics appropriate for them.