FIBONACCI NUMBERS DISCOVERED ON MARS

Dateline 2089—Shanghai Space Center. On the recent womanned space probe to Mars (cosponsored by the U.S. and China), Captain Roberta Shirra found convincing evidence of the existence of a species of flower that grew on Mars 150,000 years ago. The flower, resembling an ice cream cone in shape and only 3 mm in diameter, had small, spiked hairs on it arranged in spirals.

Capt. Shirra counted the number of spirals on the “flower” fossil using a laser scanner connected to a small microscope. She discovered that there were 233 clockwise-curved ones and 144 counter-clockwise ones. Using her background in mathematics, she realized that these are both large Fibonacci numbers.

The Fibonacci numbers have become well known in the last 20 years, thanks to the work of the Fibonacci Institute founded in Pisa, Italy in 2058. Eight years ago, the numbers 34 and 89 from Leonardo’s sequence were found to relate to the structure of organic molecules in the atmosphere of Jupiter.
MEGA-SPIRAL IN NATURE

Spirals and helices are present in many natural things. Your hair grows out of your head from one or more spiral centers (sometimes called cowlicks) usually located near the top and back of the head. The human ear is a sort of spiral. The DNA in every living cell is a helix. Even water goes down drains in a whirling helix.

But, the biggest spiral of all is the one we and our entire solar system live in—the galaxy. Our galaxy is called the Milky Way, named after a candy bar. (Actually, the candy bar came after the galaxy!)

The name Milky Way came about because of the hazy light the galaxy emits in the sky. This light can be seen on the clearest of nights. It seems to go across the top of the sky in a milky-white band that looks a bit like a wispy cloud. Astronomers have figured out that this band is really the spiral of our own galaxy as seen from inside it, where we live.

Below is a drawing of our Milky Way. A small dot and arrow show about where the sun and our solar system of planets “live” in the spiral—near the edge. The spiral of billions of stars slowly turns. If we look from Earth toward the central part of the spiral we see many crowded stars and more light in a band (the Milky Way in our sky). The band is thin because the galaxy is “thin.” (The diameter of the galaxy is about 12 times as much as its thickness.)

You might wonder, “Since there are billions of stars in it, why isn’t our inside view of the Milky Way blindingly bright?” The answer is that the stars are very far away and there is a lot of dust in space between stars, especially those crowded toward the center. This dust blots out most of the light.

Yes, our neighbors are very far away: the Milky Way is 80,000 light years in diameter. A light year is the distance a beam of light can travel in one year. This is a very long way because light travels 186,000 miles in one second.

NEW BOOK BY LEONARDO FIBONACCI RELEASED!
Author Claims It Will Change Math Forever

Dateline 1202—Pisa, Italy. Leonardo Fibonacci (“fe-bo-notch-ee”), Europe’s most famous mathematician, has just released his new book, Liber Abaci (“The Book of the Abacus”), which he claims contains “ideas that will make calculating easier than it has ever been.” The book must be ordered 2 months ahead of time because it takes that long to be hand-copied.

Leonardo, age 22, has been traveling for some years in the countries of the faraway Muslim Empire. It was in those Muslim countries that he first studied this number system as a child. He continued working with the numbers in his travels and has become quite an expert in their use.

Mr. Fibonacci suggests that Italians get used to using number symbols that look like this: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9; and he insists that the familiar-looking numbers like MMCCCXLV and CMLXXXVIII instead be written as 2,345 and 988.

One convincing reason for this bizarre procedure, he says, is that we would all then be able to add, subtract, multiply, and divide on paper, which is impossible in our present Roman numeral system. Right now the abacus, a set of wires with beads on them, is the only system we have for doing these calculations, and we can’t check our work because our calculations disappear as soon as they are done.

The story of how this amazing number system was invented, how the Muslims got it, how Leonardo Fibonacci got it, and how a number sequence got introduced by Fibonacci was investigated by this reporter and is presented on the next page for your pleasure.
HOW LIBER ABACI CAME TO BE

Leonardo, as we know, is the best and most well-known mathematician to come along in Europe in the past 600 years, about the amount of time since the Roman Empire crumbled. Recall that the Roman Empire controlled all of the countries around the Mediterranean Sea.

**Question 1:** About 800 years ago, Leonardo was a boy. What date will it be 800 years from now, when people will look back on your times the way we look back on Leonardo’s times?

**Question 2:** a) The United States was founded in 1776. How old is the United States? b) About how many times could the history of the United States have happened during the 800 years since Leonardo? c) Columbus claimed America for Spain in 1492. How long even before this did Leonardo live?

Leonardo was born the son of an Italian customs officer named William Fibonacci, in the city of Pisa, home to the famous leaning tower. Customs officers keep track of which people and what items are allowed in and out of a country’s borders, and because Pisa was near the Mediterranean, William had many shipped goods to attend to.

Little Leo attended public school and had to learn what were called the seven liberal arts: grammar, speaking, logic, geometry, astronomy, music, and arithmetic. He had to learn these in Latin, even though he spoke Italian. If he’d had this education only,Leo would not have made the big splash in mathematics that he did.

But, something happened that changed Leonardo’s fate. William was assigned to work in Northern Africa, about 600 miles across the water, in the Muslim city of Bugia—also a port city. The Christian city of Pisa had fought the Muslims of Bugia and all of North Africa in earlier times, but now they did a lot of trading with them and the other Muslim areas of Spain and the Middle East.

Europe, on the other hand, had not been part of an empire since the Roman Empire had fallen apart about another 600 years before Leonardo. Europe was just a cluster of tiny states ruled by barons who fought for power, and Pisa was a city that was its own state. It had walls around it for protection from other barons and was democratic; that is, people were able to elect their own leaders there.

**Question 3:** Put one red mark on the map where Pisa is and another where Bugia is. Draw a dotted line on the map where William would have had to sail across the Mediterranean to his new job. Color in with your pencil (lightly) all of the areas of the Muslim Empire.

**Question 4:** About what date was the fall of the Roman Empire? ______ A.D.

After a while, Leonardo’s father sent for him to come to Bugia and even hired a Muslim tutor there so Leonardo could learn some of the fabulous new arithmetic that the people were using—with numbers that were unheard of in Europe. At that time, the North Africans knew much more mathematics than did the Italians or anyone in Europe.

Leonardo quickly soaked up every math idea he could get from his tutor. Of course, he also learned to add and subtract these numbers on paper, whereas he had only done them on an abacus before. It also was much better for business than the system that shopkeepers used in Pisa, which involved the abacus and a board with compartments on it to move stones around that represented money amounts.

How did the Muslims get this system and use it for 200 years without the Europeans even hearing about it? The delay first happened because the Christians living in Europe around 1000 A.D. didn’t like the Muslims of North Africa, as their religions and cultures were different. So, for centuries, the north and south sides of the Mediterranean were like quarreling neighbors who wouldn’t talk to each other. Besides, they spoke different languages!

But, the North Africans had connections with the Middle East (whose people also were Muslims), and the Middle East had connections with the Hindus of neighboring India. The Hindus were mathematicians who had been developing a new system of numbers for about six centuries. (Everything went slower in those days!) The Arabs of the Middle East tried out the number system and really took off with it. They used the numbers for everything, adding some of their own tricks and polishing them up. They spread this system throughout the Muslim Empire.

*Continued on Page 4*
Question 5: Draw arrows on the map on p. 3 showing how the new Hindu number system spread to the Muslim Empire.

The second reason why Europe didn’t hear about it was that Europe was very disorganized after the fall of the Roman Empire. There was no central place of learning where knowledge could be shared as it had been in “the old days” at Alexandria, in Egypt. Yes, even though Egypt seems to be pretty far from Europe, it wasn’t so far by water, and that’s where the Empire had chosen to bring its smartest people and its best books together in the University and Library of Alexandria.

By Leonardo’s time, the great center at Alexandria was just a dusty memory, now a part of the Muslim Empire. Its books had been burned accidentally by Romans, then purposely by both Christian and Muslim extremists. There were no extra copies of many of these books.

A Muslim university and learning center had been started around 800 A.D. in Baghdad [in the country we now call Iraq] and it was from here that learning spread throughout the Muslim Empire, totally outclassing the knowledge in Europe.


So, Leonardo learned much about this new number system and went back to his home in Pisa, later traveling around a great deal. He learned more about the number system and exchanged ideas with the Arab mathematicians until he had mastered the subject. He even could do long division and fractions with it! He decided to write a book about it called Liber Abaci, “The Book of the Abacus” (even though the book was really about not using the abacus!).

But, what does Fibonacci have to do with a number sequence? Well, in the 12th chapter of his book, Leonardo placed a problem, among several others, about a growing population of rabbits: A certain man put a pair of rabbits [that will breed during January] in a place surrounded by a wall. Fibonacci asked: How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair, which from the second month on becomes productive?

The numbers of rabbit pairs each month turn out to produce the sequence of numbers also found on pinecones. [Editor’s Note: Of course, Leonardo didn’t know that these numbers show up on a pinecone or anywhere else. He thought of them only as the answer to the rabbit question.]

“God created the integers; the rest is the work of man.”
—Leopold Kronecker

All that most people these days know of Pisa is that it has a famous tower in it. Really, that’s one of its least important accomplishments.

As the article on Leonardo Fibonacci explains, Pisa is an Italian city. Once the walled “capital” of a territory in northern Italy, it was made up of soldiers and sailors who defended it. And, it was a very wealthy city, as well.

For a major city project in 1174, the Pisans decided to build a handsome tower in the town square near a cathedral and a cemetery. About a year later Leonardo Fibonacci was born.

The Pisans built the tower with eight tiers of round, arched arcades with a spiral staircase in the middle. They put white marble around the outside. In all, it looked like a large birthday cake.

But, the construction was no piece of cake—the top of the bell tower, 180 ft. above the ground, wasn’t finished until 1350. The tower was very heavy with all of its adornments. It began to lean slowly. Many feared it would fall over.

It now slants more than 5°. That doesn't seem like much but it looks very slanted. Yet here it is, 650 years later, and it still hasn't fallen. The ground was reinforced underneath it to keep it from leaning farther, but nobody wants it to stop leaning—it's too famous that way.

Question 1: How many years did it take to build the tower?

Question 2: About how many stories high is the tower (if a story is around 9 ft.)? Imagine a building you’ve seen that has about that many stories. Suppose it’s leaning. Would you think it was going to fall soon?
Across
4. A number divisible only by itself and 1.
5. A comparison of two things by division.
8. A flower necklace in Hawaii.
9. Each Fibonacci is the _____ of the two previous ones.
10. A job that Eratosthenes did in Alexandria.
12. The best way to see how close Fibonacci ratios come to becoming the Golden Ratio.
15. A number is ______ by a number if the first is a multiple of the second.
16. Several spiral springs.
20. A tree that likes to make Fibonacci Numbers.
21. 564 is a ______ of 2.
23. Nationality of Eratosthenes.
24. First two letters of name of body of water Leonardo sailed on.

Down
1. Circumference means distance ______ a circle.
2. The Hindus and Arabs developed a number system based on _______.
3. For three consecutive numbers of the Fibonacci sequence, the product of the outer ______ differs by 1 from the square of the middle one.
4. Leonardo was born there.
5. Decimal numbers are easier to ______ than Roman numerals.
6. Leonardo’s father, William, worked there.
7. If on a “Greek face” the distance from eye level to ______ is divided by the distance from eyes to nose bottom, the result is the Golden Ratio.
8. The first letters in “Left” and “Right.”
10. The riches and learning of the Middle East were mostly unknown to Europeans, who called the area a strange _______.
11. The first letters of “Bee Hive.”
13. A tasty object with Fibonacci Numbers “decorating” it.
17. A curved figure you can make from the Golden Rectangle.
18. If you divide the distance from _____ to sternum by the distance from the sternum to the top of the head, you get the Golden Ratio.
19. Eratosthenes’s calculation of the circumference of the Earth was very close to the ______ value.
21. Abbreviation for a tenth of a centimeter.
22. If you divide the distance from knee to sternum by the distance from knee to ______, you get the Golden Ratio.
THE WAYS OF THE HIVE

Bees live and work in a kingdom that sounds more like science fiction than reality. For instance, did you know that bees were originally tropical insects? Now they live all over the world, so they try to make themselves feel at home in colder places by clustering into a big ball and keeping their wings flapping whenever it’s colder than 57°F. They do this until it gets to be a balmy 90° in the ball!

There is no aircraft anywhere that can transport loads much heavier than the plane itself, but a bee can. It loads pollen into the pollen baskets attached to its legs and also sips nectar until the bee is much heavier than twice its usual weight. (Sometimesumble bees get so heavy they can’t even take off!)

In a typical beehive there are 35,000 to 50,000 bees, all living harmoniously.

Question 1: What town in your area has about this many inhabitants? If that town were like a beehive, each person, on the average, would only have about 2 cubic meters of space. This would require about 80,000 cubic meters of space for everyone in the town. If this much space were a large box, like a hive, the box would be about 43 meters in length, width and height. (43 x 43 x 43 = 79,507 cubic meters.)

Question 2: A city block is about 80 meters on each edge. Draw a picture of how this city-beehive of people would look standing on a square city block. Imagine that! The whole town you named in Question 1 living in less than a city block—that’s how the bees live.

Question 3: If a bee is 1/2 inch long and there are about 300 feet in a city block, about how many blocks long would a “parade” of 40,000 bees from one hive be (with no spaces between bees)?

FIBONACCI SOCIETY STARTED!

Dateline 1962—Los Angeles, California. About 80 years after the work of Edouard Lucas, someone has stepped forth to start a Society of the Study of Fibonacci Numbers. This person is a Catholic brother in the order of Christian Brothers teaching at Saint Mary’s College in Los Angeles. His name is Brother Alfred Brousseau.

Brother Alfred has extensively investigated the Fibonacci Numbers first studied by Edouard Lucas in the last century. He has found that they are present in pinecones, sunflowers, and many other natural objects. He has invited mathematicians and scientists from all over the world to send in articles about their findings concerning these numbers, and will publish them four times each year in his Fibonacci Quarterly.

BEES FORM LABOR UNION AND UNIVERSITY

The bee city has three types of members. There is one female ruler, the Queen Bee. She rules over a population that is mainly female bees, who do all of the work. These female bees are called, of all things, “workers.”

There are male bees too, called “drones,” with large eyes and larger bodies. But, there are only a few hundred of them and they have only one job. They have to breed with the Queen to fertilize her eggs. After this, they stick around for a few days and then die.

The thousands of workers have many different jobs. The young “teenagers” (about 14 days old) build honeycombs by eating lots of pollen and honey. Then they let wax come out of ducts in their bodies. They form the wax into perfect hexagon-shaped cells, where new “baby” bees will snooze as they develop for about 10 days before they’re ready to get busy. [Editor’s Note: See the hexagon game that these bees like to play on page 7.]

When the teenagers become about 21 days old, they are “adults” ready for the more heroic job of going out and looking for nectar and pollen to feed everybody. Lots of food is needed. For instance, each baby bee has to be fed about 1,500 times every day! And, all of the 34,000 workers eat a lot of pollen

WEIRD FRACTIONS WITH ONES

\[
\frac{1}{1+1} = \frac{1+1}{1+1}
\]

The above fraction has something to do with Fibonacci Numbers. Simplify it and see:

What do you get? Continue the pattern a couple more steps, and then simplify the fraction to get a surprise! If you’re really ambitious, continue more patterns and continue to be surprised.
and nectar just to keep flying—though fortunately, most eat “on the run” while they work. Then, there is all that wax that has to come from their bodies.

**Question 4:** Assuming baby bees get fed during all 24 hours of the day, calculate about how many minutes there are between each feeding.

Workers live for about 6 weeks, so they try to get everything done quickly. And, they can’t mate with a drone (only the Queen can do that), so they pretty much always have their minds on their jobs. One of the more respected jobs is to be a scout who looks for new flowers. She flies for miles and comes back with the exciting news, which she tells to the other workers by doing a dance. The steps draw a diagram of how the sun in the sky will appear to move as the bees fly to the flowers. After her dance, many gatherers fly right to it.

A job that’s not quite so flashy is to be part of the crew that hovers near the hive’s entrance so their wings will fan the smells of the hive out and the returning bees can hone in on the smell.

**Question 5:** Assuming the Queen works for 18 hours each day, approximately how many minutes does it take to lay each egg?

**Question 6:** How many worker bee lifetimes make one Queen lifetime? (Remember, there are 52 weeks in a year."

So, how does a worker get to become Queen? The Queen is selected from birth, and just like human queens, she is given special treatment right away. The workers have glands in their heads that make a very nutritious goo called *royal jelly* that is fed to all babies for their first 2 days, but to the Queen all of her life. This jelly changes her bodylines to be longer and makes her capable of breeding and laying eggs. She even has the ability to lay either a fertilized egg or an unfertilized egg by choice (see below!)

One of the Queen’s other features is that a substance comes out of her head (kind of like bee chewing gum) that she gives to the workers to pass out to each other. The workers like it and chew on it. But, this gum is powerful—it has special chemicals in it that make all of the workers more aware of the jobs they’re supposed to do and makes them willing to do anything to serve the Queen.

The Queen lives for about 3 years, and her only job is to lay eggs. She gets so full of eggs that they weigh more than she does! She has to lay about 1,700 of them a day and each one requires special attention. She has to wiggle her back end into a small wax cell (made by the teenagers) and give off one egg that sticks to the bottom of the cell. Needless to say, workers help the overloaded Queen waddle along.

**HEXAGON GAME**

Here’s a game that the bees like to play in the 35,000 honeycomb cells of their hive. Each cell is shaped like a perfect hexagon. Remember that a hexagon is a figure that has six equal sides. Notice how well hexagons fit together!

The 2 playing boards below are for 2 separate hexagon games. Have a friend play with you.

**Rules**

1. One player picks the two A sides of the board for goals while the other player picks the two B sides for goals. Players pretend they’re worker bees.
2. Using a different-colored pen, each player takes turns coloring one hexagon at a turn, pretending they’re building these cells with wax.
3. The first player to complete a chain of “wax” hexagon cells from one goal to the other wins.
ERATOSTHENES MEASURES THE EARTH!

Dateline 2231 B.C.—Alexandria, Egypt. Eratosthenes (air-ruh-toss-think-knees), a man of many talents from Greece now living in Alexandria, has just measured the circumference of the Earth! Rather than using a ruler he used an ingenious method involving shadows on the longest day of the year.

He has concluded that the Earth’s circumference is 250,000 stades \[\text{Editor’s Note: This is about 25,000 miles, which agrees very much with the 20th century value of 24,901 miles.}\]

**Question 1:** How many years ago was 231 B.C.?

**Question 2:** How long, compared to a mile, is one stade?

How Eratosthenes made this measurement, how he got to Alexandria from his native Greece, and his other magnificent accomplishments will be described in this article.

First, the measurement, his best “trick” yet: Eratosthenes measured the Earth by using a deep well that is known in the city of Syene, 5,000 stades south of Alexandria. Something interesting happens in the well on the longest day of each year, the day of the Summer Solstice. \[\text{Editor’s Note: The Summer Solstice occurs on June 21st on modern calendars, although June’s name wasn’t invented yet in 231 B.C.}\]

At that time in Syene, the sun is exactly overhead, which means no shadows are cast by anything that stands upright. It means that the whole floor of the well is totally lit by sunlight for a few minutes.

There are only a few places on Earth where this happens. They all have to be located on the tropic of Cancer, it turns out, because this is the circle on the Earth that the sun stands above as the Earth turns on the longest day of the year. This well was help for Eratosthenes’s experiment because it gave him an exact place and time on the Earth that the sun was directly overhead.

The clever Eratosthenes measured the angle a shadow made with a building that was standing straight up in Alexandria. He discovered that the sun was tilted at the angle of \(\frac{1}{50}\) of a circle away from being straight overhead.

**Question 3:** How many degrees is \(\frac{1}{50}\) of a circle, when a whole circle has 360°?

He then said that if a 5,000-stade trip north from Syene to Alexandria makes the sun appear to shift \(\frac{1}{50}\) of a circle, then the sun would shift a whole circle—360°—if he traveled 50 times that far north, because 50 times \(\frac{1}{50}\) is one whole circle.

But, that would be clear around the Earth over the North Pole and back to Egypt. And, 50 of those 5,000 stade trips makes 50 x 5,000 = 250,000 stades, and that must then be the circumference of the Earth!

**Question 4:** Let something be a sun over Egypt again and let your finger stand on Egypt (on the globe) like a person. Let it travel, standing straight up on the globe always, to the North Pole, over the Pole, south, and back around the South Pole to Egypt. Now do it again, letting the little person (your finger) look at the sun all the while. Pretend the person can even see the sun through the Earth. How does the position of the sun appear to move for that person?

**Question 5:** Remember that Eratosthenes’s 250,000 stades is 25,000 miles. On a world globe, locate the equator. How many miles do you think it is across the Pacific Ocean at the equator? The Atlantic Ocean at the equator? \[\text{Editor’s Note: The fact that the ancient mathematicians were solving questions like this could be surprising if you recall that many people in Europe a thousand years later still thought the Earth was flat! How do you think this important piece of information was forgotten?}\]

[In the article about Leonardo Fibonacci’s book, read about how Europe fell behind the Muslim countries in mathematical knowledge, and you’ll know how this fact became unknown. Thus, it happened that in 1490 Christopher Columbus had to persuade Queen Isabella of Spain that the Earth was round!]

Eratosthenes also is famous for being a poet, historian, athlete, and mathematician. He was invited to Alexandria by our Pharaoh, Ptolemy, to become a private teacher for his son. Eratosthenes lived in the great learning center of Alexandria, and he has just been appointed to be the librarian of Alexandria’s famous university. \[\text{Editor’s Note: Little did he know that}\]

Continued on Page 9
Dateline 2028—New Delhi, India.
A new metal has been created in the high-energy cyclotron of the Institute for World Cooperative Science in New Delhi. Doctors Lief Delquist and Antonio Maro set the atom smasher on highest energy and watched as tiny atoms of the new metal, which they named fibonaccium, were blasted into existence. The atoms traced the expected curves in a cloud chamber.

What’s so special about fibonaccium? First of all, it is the heaviest metal known to exist. It is an element, which means its tiniest particle is an atom.

(Non-elements are called compounds and their smallest particles are molecules, which are several atoms of different elements stuck together like tinker toys. For instance, sugar has 1 carbon, 12 hydrogen, and 6 oxygen atoms joined in a kind of ring.)

Up to now, there were only 109 known elements—and only 92 of these are found in nature, making up every known thing in the universe. The other 16 have been created in laboratories and tend to fall apart with time, most disintegrating in a fraction of a second.

Experiments have shown that fibonaccium is stable, that is, it can continue to exist without splitting into two more comfortably sized pieces. This is remarkable because fibonaccium is a “gigantic” atom, fitting in the periodic table of elements at the 114th position, way past the big elements that easily fall apart. Scientists had predicted, however, that for a lot of reasons an atom with exactly this size should be stable.

This 114th element has 114 protons and 184 neutrons in its nucleus. Its neutron/proton ratio is 1.614, which is extremely close to the number known since ancient times as the Golden Ratio. Because a series of numbers called Fibonacci Numbers is known to make Golden Ratios, the scientists felt it was fitting to name the new element after the brilliant 12th-century Italian mathematician Leonardo Fibonacci.

The smaller, lighter elements don’t have this ratio between their neutrons and protons. Here are some examples:

<table>
<thead>
<tr>
<th>Element</th>
<th>Protons</th>
<th>Neutrons</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Carbon</td>
<td>6</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Cesium</td>
<td>55</td>
<td>77</td>
<td>1.4</td>
</tr>
<tr>
<td>Uranium</td>
<td>92</td>
<td>146</td>
<td>1.58</td>
</tr>
</tbody>
</table>

The smaller, lighter elements: hydrogen, carbon, cesium, and uranium have ratios far from the Golden Ratio. However, the heavier elements have ratios close to the Golden Ratio. For example, uranium has a ratio of 1.58, which is close to the Golden Ratio of 1.618.

The ratio of the weights of strontium and neodymium is very near the Golden Ratio. Strontium has a weight of 87, and neodymium has a weight of 147. The ratio of the weights is 1.693, which is very close to the Golden Ratio.

Question 1: Describe what happens to the neutron/proton ratio as elements get bigger and heavier (mention the Golden Ratio in your answer).

Another Golden Ratio curiosity is that the uranium atom shown in the table above is radioactive—that is, it tends to fall apart and blast out a tiny packet of heat energy as it does so. It also shoots out a neutron, which can hit another atom and make it fall apart. If there is enough uranium around (called a critical mass), all of the atoms will start shooting it out and falling apart, releasing gigantic amounts of heat—this is what makes the atom bomb work.

Now, when uranium falls apart and loses a neutron, it creates two smaller atoms, generally strontium and neodymium. Strontium has a weight of 87 (protons and neutrons together), while neodymium weighs 147. The ratio of the weights of the two pieces is very near the Golden Ratio.

Question 2: Find the ratio of the weights of strontium and neodymium. How close to the Golden Ratio is it?

Question 3: Why do you think uranium does not prefer to break in half, but instead splits toward the Golden Ratio?

600 years later, after the fall of Rome, this wonderful center of learning would lie almost deserted and in ruins, its books burned.

The mathematical idea that bears his name is the famous “Sieve of Eratosthenes.” (A sieve is a strainer.) It is a simple method for figuring out which of the first 100 counting numbers are prime numbers. [Editor’s Note: A student activity page will show you how this is done.]
Dateline 2015—New York. A survey of 13,000 scientists from the Cooperative Science Council of the World has been completed. It shows that the mathematical tool most popular among scientists is the ratio. A survey of 2,100 money advisors and 375 artists found the same thing. Dr. Jean Peepers of the Relativity Institute said she was happy to see the ratio win the competition for “Most Popular Tool” because she uses ratios in her laboratory in every step of her work.

She added, “Just today, I needed to compare the number of mice that could levitate with the number of cats that could. (I work with anti-gravity.) I have 72 floating mice and 24 ‘flying’ cats in the experiment. And the mice leave the ground 2 ½ times the speed that the cats do.

“So, first off, this means the mice are in the ratio of 72 to 24, or 72⁄24, or, if we reduce it, 3 to 1 with the cats. This tells me that every cat can be thought of as being grouped with three mice (though they never are actually in contact with each other). This is the easiest way for me to think of how the mice outnumber the cats.

“I also know that the ratio of the speed of the mice in leaving the ground against gravity to the speed of the cats is 2 ½, meaning that the mice are a lot faster to float. In fact, this means they lift 2 ½ cm as each cat lifts 1 cm, or 2 ½ inches as each cat lifts 1 inch.”

The artist Alex Spredder paints Martian landscapes based on photographs from the last womanned U.S.-Chinese expedition to Mars. He agrees that ratio “takes the cake.” He adds, “When I’m trying to mix an outrageous red-orange for a Martian dust-stormy sunset, I stir 4 lumps of yellow with 16 lumps of red. I create a mixture whose yellow-to-red ratio is 4 to 16, or ¼. Because ¼ reduces to ½, I know that there is ½ as much yellow as there is red in my orange sunset. This ratio also tells me that for every 1 lump of yellow in the mixture I use 4 lumps of red.”

Here’s more about how the people surveyed said ratios work. Mary Goldstone kept track of the funds for the last chess-in-space game in the Japanese space shuttle mini-auditorium. She puts it this way: “A ratio means comparing two things by division.

“We can compare two amounts that have the same units, like $12 for a laser stopwatch and $3 for a two-way radio. We say the prices are in the ratio of 4 to 1 (because 12 ÷ 3 = 4, and 4 = ¼), which means that there are four dollars in the stopwatch price for every one dollar in the wrist radio price. That means every $4 of the $12 price has the $1 assigned to it from the $3 price like this:

$12: $$$$ $$$$ $$$$ laser stopwatch
$ 3: $ $ $ wrist radio

“We also can compare two amounts that don't have the same units, as Dr. Peepers did in comparing cats with mice. Then, we get answers like this: 72 mice/24 cats = 3 mice per cat, or 3 mice/cat. Remember, this means that each cat pairs with three mice.”

[Editor's Note: In Unit III you usually will compare two things that have the same units, like 34 cm and 55 cm. You then will be asked what the ratio between them is; this ratio is stated as a fraction or decimal with no units.

[Division of the first number by the second always finds the ratio, even if the first number is smaller than the second. This may require a calculator if you want to do it fast. For instance, 34/55 = .61818, or .618 rounded off.

[The meaning of this ratio .618 is that 34 is .618 the size of 55. How big is that? Because .6 means 6/10, we conclude that 34 is about 6/10 as big as 55. So, a ratio really describes how big one number is as a part of the other number. That'll be very handy, as you'll see.]

Question 1: Find the ratio (in lowest terms) of the following: a. 36 boys to 20 girls b. 88 cm to 102 cm c. 64
GOLDEN RATIO: IN STYLE FOR YEARS

It’s true that the Golden Ratio, the number 1.618 . . . , is in a lot of unexpected places in nature. But, is this why the ancient peoples loved it so?

This question is hard to answer. The Golden Ratio first may have been loved for its mathematical interest. But, there is evidence that it also may have been known as a number relating to the human body many centuries before Christ. Read the evidence below. Then, decide your answer.

Much ancient Egyptian architecture has been shown to have the Golden Ratio in it. Later, you will learn how appears in the Great Pyramid of Cheops. But, can it possibly be that the Golden Ratio just shows up when shapes are carefully made to be balanced and beautiful?

A Frenchman named Rene Schwaller, who spent years poking around Egyptian ruins and pictures, has this argument:

"Occasionally one hears this remark: 'The Golden Number can be found everywhere.' This is incorrect. But, if practice and usage have established a balanced form that is pleasing, or . . . stable . . . the Golden Number may be sought there. No one thought of this number in establishing [art form], any more than a plant would think of it, or a [pregnant] woman carrying a child."

Question 1: State in your own words the idea that you think Mr. Schwaller is expressing. Say it in one or two short sentences.

But, Rene Schwaller also found that a whole temple was made by the Egyptians so that its rooms were placed as parts of the human body. He found the Golden Ratio in that temple’s measurements and pictures. Couldn’t the Egyptians have known for thousands of years that the Golden Ratio was in the human body?

Question 2: What do you think?

If the answer to the above question is really yes, then maybe this is the first reason why the Golden Ratio was so important in ancient times: It is part of the plan of our own bodies.

But, the rest of this tour will show that the Golden Ratio is a mathematical marvel, as well. So, its mathematical, geometrical beauty also may account for its popularity.

There may be a third reason why it’s popular. The Golden Ratio has to do with division of things into two unequal parts. It gets boring to divide things into two equal parts, especially when you’re doing artwork. Still, things have to be interestingly unequal.

It’s been found by artists like Leonardo da Vinci that very beautiful things result when you place a dot that divides a line into two parts, as seen to the left. Here, the ratio of the big part to the small part is 1.618 . . . And, a bonus this ratio gives is that the ratio of the whole line to the longer piece is also 1.618 . . .

Many artists over the years have made sure that when they paint or draw, the lines they make divide other lines at the Golden Ratio point, rather than just in half. Nature does this also in the design of shells and in splitting atoms. (See the article "Fibonaccium Discovered" in this Tour Guide newspaper for more about atoms.)

As an example, pictured to the left are an ancient Greek vase and a typical picture of a bird’s head created by a Northwest Native American.

Question 3: Give the measurements of the two line pieces to the left and the whole line: _____cm, _____cm, and _____cm. Find two ratios they have with each other that are close to 1.618.

Question 4: Where is the Golden Ratio in each of the pictures? Measure many different lengths and check for the Golden Ratio among them.

Question 5: Which of these artists, do you think, worked with the Golden Ratio in mind? Explain your answer.

EDOUARD LUCAS NAMES NUMBER SEQUENCE AFTER FIBONACCI

Dateline 1877—Paris, France. Leonardo Fibonacci was a Pisa, Italy, mathematician who in 1202 A.D. introduced a certain problem about the reproduction of rabbits in a book called Liber Abaci. A little-known group of numbers comes from solving that problem, and for 675 years until now no mathematician has ever commented on them.

Finally, mathematician Edouard Lucas has just written a book about sequences of numbers. In it he has mathematically investigated the sequence 1, 1, 2, 3, 5, 8 . . . obtained while solving Leonardo Fibonacci’s famous rabbit problem.

Mr. Lucas has chosen to call that sequence the “Fibonacci Sequence” in honor of Leonardo. [Editor’s Note: Many of Lucas’s tricks with the numbers will come out in your student activity pages.]
MORE HISTORY OF THE GOLDEN RATIO

Here are some steps in the evolution of the Golden Ratio that we can get from the historical record. There are many steps still unknown, of course.

The Rhind Papyrus, an old parchment dated about 1700 B.C., was found in the wrappings of a mummy in 1855 A.D. It was like a math book with many problems solved. It said there was a certain “sacred ratio” used in building the pyramids. (This tour will take you to the Great Pyramid to see how this was done in Unit V.)

The Pythagoreans (about 600 B.C.) knew that the Golden Ratio had to do with measurements of the body. They studied these measurements. See “Secrets of Pythagoras’s community Exposed” in this newspaper for more about these remarkable people.

The Parthenon (about 440 B.C.) was the best building the Greeks could build. They dedicated it to all of their gods. They felt that several golden ratios would make it more worthy so they built these in. Its length and height make one of those ratios.

Plato (around 350 B.C.) studied everything he could find written by Pythagoras. He said the Golden Ratio is the “most binding of all mathematical relations, the key to the physics of the cosmos.” (See the article “Plato Pulls New Surprises” elsewhere in this newspaper for more about this man.) Then, there was a long gap in the interest in this ratio. Even Fibonacci, whose numbers contain the Golden Ratio (as you will see in Unit IV), didn’t really take note of it.

Pacioli (1500 A.D., Italy) wrote a book called The Divine Proportion with beautiful pictures done by Leonardo da Vinci, the brilliant artist of the Renaissance. It was all about the Golden Ratio. After reading this book, many other artists started using the Golden Ratio in their paintings. This has continued all the way up to the modern day.

Le Corbusier, a famous architect of the early and mid-20th century, has used the Golden Ratio in the design of many buildings, including the United Nations Building in New York.

Question 6: Where is the Golden Ratio in the United Nations Building shown below?

Question 7: Draw a simple object, making sure you have the Golden Ratio in it in at least two places.

THE MAGICAL PENTAGRAM OF OLD

Civilizations often use symbols to stand for a certain very large ideas. The Christian cross and the Nazi swastika are two examples from our more recent times. The pentagram, too, has been used this way in the past.

Many older civilizations regarded it as a power symbol, although different ones had different ideas about what kind of power it had. In ancient Babylonia, 1000 years before Christ, the pentagram was considered to be magic, sometimes of a very negative kind.

The Pythagoreans, 600s B.C. (see the article about their secrets in this newspaper) loved this symbol, using it as a kind of secret stamp, and felt that it represented well being, health, and “the wisdom of going between extremes.”

The Druids (100s B.C.–200s A.D.) had some uses for the pentagram. They were a secret group that lived mainly in England and Scotland. Theirs was pagan Earth-religion. They believed that old, gnarled oak trees have lots of Earth power, as do wells and stones arranged in a circle.

However, the Druids didn’t build the great stone circles like Stonehenge in England. Ancient primitive peoples living more than 1000 years before the Druids dragged huge stones long distances to build them. However, it is believed that the Druids used Stonehenge and other circles for their rituals, which sometimes included blood sacrifices and the pentagram to focus Earth and sky energy. Although the Romans also were pagan in their religious beliefs, they wanted to blot out the druids.

A German word for the pentagram is Drudenfuss (drood-en-foos), which means “the foot of a Druid.” Around Druidic times people believed that when the pentagram was turned upside down it looked like a horned goat and represented evil.

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Dear Goldie,
I’m about to turn 13 and my friends say 13 is an unlucky number. I have to be 13 for a whole year and I don’t want to be unhappy that long. What can I do?
—Preteen

Dear Preteen,
The number 13 used to be considered unlucky but now you know that 13 is one of the beautiful Fibonacci Numbers. Tell your friends you are finally coming up to a “Fibonacci Birthday” after having your last one at 8. You won’t be able to celebrate like this again until you’re 21. Put pinecones on the cake and serve pineapple too.
—Goldie

Dear Goldie,
I’m having trouble sleeping at night. I have too much homework and my boyfriend is starting to look at other girls who look more rested up. How can I get some sleep?
—Wreck

Dear Wreck,
You need to be sleeping on a more perfect bed. Ask your parents to buy you one that’s exactly a Golden Rectangle in shape. You’ll dream of seashells, mountain goats, and galaxies. Shop for your bed with a tape measure and a ruler. Sweet dreams!
—Goldie

Dear Goldie,
I want to give my friend a gift for the New Year, but I’m stumped. He needs a calendar, but they’re boring.
—Stumped

Dear Stumped,
How about making 12 large, perfect pentagons, and on each one draw a calendar for a month. Assemble these to make a dodecahedron and give it to your friend wrapped like a ball. Or, if you are more adventurous, make 20 triangles. On 12 of them, put months and on the other 8, put pictures or funny sayings. Assemble all of these to make an icosahedron.
—Goldie

Dear Goldie,
I can’t believe the Egyptians knew that the Earth was round and that the Earth and moon make a pyramid shape. They didn’t have the technology that we have today to measure these things.
—Skeptical

Dear Skeptical,
We don’t know what things the Egyptians actually knew but we are certain of one thing: They knew far more than we used to give them credit for. They may have used ideas and theories not even attainable with our kind of technology.
—Goldie

The pentagram lost some of its meaning of power or magic in the Middle Ages. It came to mean “man” in the sense of “humanity.” The star even looks like a person standing with arms outstretched.

Many Renaissance painters, such as Leonardo da Vinci, Rafael, and Fra Lippo Lippi, used the pentagram as a background diagram or grid, for placing people and things in a picture in a meaningful way.

When the United States was founded, there was new interest in the pentagram. Pentagram stars were selected by the founding fathers to adorn the U.S. flag—each one representing a state. This was not just a coincidence: There is strong evidence that the founding fathers were students of ancient and mysterious mathematical ideas.

On the Great Seal of the United States (see the back of a dollar bill), 13 pentagrams (representing the original 13 states) are arranged in the form of a six-pointed star. From ancient times, the six-pointed star, also chosen by the Jewish people as their symbol, meant the mixing of that of the Earth. The other side of the Great Seal, also on the dollar bill, shows an eye over a pyramid, a symbol for wisdom that dates back thousands of years.
Dateline 387 B.C.—Athens, Greece. Greek philosopher Plato has just announced the opening of his Academy. It is a school devoted to the study of music, arithmetic, geometry, astronomy, philosophy, and science. But, it also is a place where very important scholars carry on research. In this exclusive interview Plato tells why he started the Academy and answers some key questions posed by this reporter.

[Editor’s Note: Twelve years before this interview, Socrates, Plato’s teacher, had been forced to drink poison by the authorities because of his controversial ideas. Plato, his most famous student, had to run away to hide for a few years. Later, he came back and founded the Academy.]

**Reporter:** Mr. Plato, what’s so important about music that you insist on having its study in your Academy?

**Plato:** Music is made up of vibrations, and the whole universe is vibrating all the time. There are very slow vibrations, such as when the planets seem to move forward in the sky and then backward from night to night.

**Reporter:** So, music and astronomy are sort of the same thing?

**Plato:** Correct—Pythagoras called this planetary movement, “The Music of the Spheres.” I respect his thought very much.

**Reporter:** I guess you liked Socrates quite a bit too?

**Plato:** Yes. Even though he took some very unpopular positions on things, he was my teacher.

**Reporter:** Some say you are a great mathematician.

**Plato:** No, I’m not, but I like to hang around with great mathematicians and learn from them. And, I seem to be able to express their ideas better than they can sometimes, so people think they’re my ideas.

**Reporter:** I understand you really like certain numbers.

**Plato:** Oh, yes, like the number 5,040, for example. I like 5,040 because almost anything will divide into it (except 11). I feel that the ideal city would be one with 5,040 families and houses so they could be divided up perfectly into neighborhoods, clubs, and political groups.

**Reporter:** And, you feel the same way about certain solid objects?

**Plato:** What? Oh, yes, you mean the five perfect solids. They tell the story of our universe. I have a sort of diagram that explains this.

Let me describe it. There are four qualities or elements that make up everything by mixing together in different amounts. These are Fire, Earth, Air, and Water. Somewhere in the world of perfect ideals, each of these perfect qualities exists in everything, but on our imperfect Earth they mix imperfectly. For instance, a tree has a lot of juice (Water), some hardness (Earth), vapors (Air), and the energy of growth (Fire).

In the diagram, Fire is the opposite of Water, and Air, which makes up the sky, is the opposite of Water. The simplest solid, the tetrahedron, represents the essence of Fire, which is opposite the most complex, the icosahedron, Water.

The cube, Earth with its four directions, is opposite the octahedron, Air, because the cube has six faces and eight vertex points while the octahedron has eight faces and six vertex points.

Meanwhile, the dodecahedron represents the whole universe because its 12 faces represent the 12 astronomical signs of the zodiac that ring our sky and everything in it.

These solids are everything to me.

**Reporter:** I believe you. Thank you for this time away from your busy teaching schedule.

[Editor’s Note: The number 5,040 will turn out to be important for another reason. You will see this toward the end of Unit V. After Plato’s death, his thought influenced both church and state in Europe for more than 1,000 years. Today his ideas are considered a work of genius, even though many are out of date.]
Editor’s Note: This article was written by a reporter who lived as a spy in the Pythagorean Community at Croton, which is at the ball of the foot of Italy’s boot. Italy was called Greater Greece then, so it was a Greek community. The Pythagoreans (pronounced pi-thag-o-ree-uns) are named after their notorious leader, Pythagoras (pi-thag-o-rus).

Different people have called Pythagoras an astronomer, a sorcerer, a prophet, a mathematician, a miracle worker, or a fake.

Some have called the community a destructive cult, some a school, some a gathering of friends. They were very secretive, and their neighbors were not sure what they were up to. A year after the reporter was there the group got run out of town by a mob, and some were hanged. Pythagoras escaped. It is hoped that this article will clear up some of the mystery of who they were.

Dateline 535 B.C.—Croton, Greece.

I didn’t know what to expect when I went to live with the Pythagorean Community. I was very interested in numbers and had heard that their motto is “All is Number.” Their secret symbol is the five-pointed pentagram star, which has the Golden Ratio in it, and I wanted to learn more.

I knew this group was different from most of the religions that are around these days. They weren’t like the popular Dionysians, who have wild parties as their way of worshiping their god, Dionysus. I’ve heard of Buddha, who has been teaching “the Middle Way” in India, and Confucius, the wise man of China, but Pythagoras seems different from them. He loves numbers and thinks they are keys to the secrets of the universe.

He uses a strange new word, mathematics, which means, “desire to learn.” I had to begin studying their four required subjects: arithmetic, geometry, music, and astronomy. The Pythagoreans think these subjects are somehow all the same. For instance, the planets move through the skies in a repeating rhythm that they call “the music of the spheres.” The bodies we see moving in the heavens are seven in number, and they think of the number seven as meaning “a stage of completion,” so the heavens are “at a stage of completion” to them.

Question 1: What is a sphere?

Question 2: What are those seven moving bodies that are visible to the naked eye? The Pythagoreans have named the seven days of the week after those seven heavenly bodies.

Question 3: What three days’ names still relate to heavenly moving bodies?

Numbers are like persons to the Pythagoreans. Some pairs of numbers are “friendly,” the odd numbers are “male,” the even numbers are “female,” and some numbers are “perfect.” (For instance, 6 is perfect because 1, 2 and 3 are all that divide into it and 1 + 2 + 3 = 6.)

Question 4: Why is 28 a perfect number?

The community doesn’t serve meat because we might be eating our great-grandparents! They believe that when a person dies his soul may return in an animal. If my great-grandparents came back as cows, they might be my dinner steaks! When I can’t eat meat, I like to eat beans, but they don’t allow that either (especially lentils), for some reason. I just stick with bread and cheese.

Because of these weird customs, the Pythagoreans aren’t too popular in Croton. They hold top-secret meetings. The only way to get in is to have their beloved pentagram on you in the required place on your body, like a hand.

Most people think our number system is based on 10 because we have 10 fingers. The Pythagoreans feel that 10 is more important than our fingers. Ten is 1 + 2 + 3 + 4. But 1 means “unity” or “source,” 2 means “difference” or “balance,” 3 means “harmony,” and 4 means “justice.” So, 10 represents what they think of as “The All.” They have a lot of reverence for the tetrahedron, too, because it has six edges and four faces, or 10, as its “Number.”

They practically worship the other perfect solids as well. These are the cube, the octahedron, the icosahedron, and the dodecahedron. Each has perfect triangle, square, or pentagon faces. The dodecahedron’s pentagons all have pentagrams hidden in them, of course, so the Golden Ratio is everywhere in that solid. They like that one the most.

There is some dark secret they have discovered but won’t talk about. It’s something about the fact that if you have a square with sides 1 cubit long, the length of the diagonal can’t be expressed as a cubit plus an exact fraction. They believe every number in the universe is either a whole number or a fraction (ratio). That would mean that the length of the diagonal is “irrational.”

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PYTHAGOREAN PUZZLE

(Needed: Ruler and calculator)

Here’s a right triangle. (A right triangle has a right angle, that is, a square corner.)

A square is on each side of this triangle. The ancient Greeks called this squaring the sides.

Measure each square to the nearest millimeter and compute its area. Write the area inside each square. Two of your numbers should add up to the third. Do they? Remeasure and recompute if they don’t.

The famous “Theorem of Pythagoras” says this will work on any right triangle. This fact was really known way before Pythagoras’s time, by the Babylonians and Egyptians. There are so many things Pythagoras invented but didn’t get credit for that he deserves to have this named after him anyway!

Make your own right triangle with sides of different lengths and try this area trick again.

An advanced insight [Unit V]: A right triangle of sides 3-4-5 was loved by the Egyptians because all sides were small whole numbers in sequence.

Notice that the squares of the two shorter sides add to the square, 25, of the long side.

Find your Earth-moon-pyramid drawing from the “Golden-Pyramid Payoff” activity. Draw a square around the moon. Draw slanted “shoulder” joining top corners of the Earth square and moon square. Think of 720 miles as “one unit” of length—call this unit a jump. Then, how many jumps does each side of the right triangle have? Would the Egyptians be happy with this “shoulder” triangle?

Question 5: Mathematicians know that the length of that diagonal would be $\sqrt{2}$ cubits = 1.414 . . ., an endless string of digits that never repeat in a pattern. An irrational number is this way, and that’s considered just fine. To see more digits, put 2 on your calculator screen and then push the $\sqrt{}$ button. Write the digits: _____ Does $\sqrt{3}$ look irrational? _______

SECRETS

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n’l” (not a “reasonable” or “ratio-able” number). And, that would blow their whole idea that “All is Number.”

BRAIN BENDER

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Alicia planted 10 cabbages in her garden so that she had them in five rows of four cabbages each! How did she do it? Try drawing your answer.

INSIDE THE GREAT PYRAMID OF CHEOPS

Dateline 820 A.D.—Baghdad, Persia (Iraq). Prince Al Mamun, son of ruler Harun al Rashid, has just returned from Egypt on an expedition to explore the Great Pyramid there. We have interviewed him and constructed this story of the adventure.

The Great Pyramid is a huge, white-pointed silhouette in the Egyptian desert—visible for miles. The Prince and his caravan of horses and camels trudged through endless sand dunes to discover its secrets.

At this time the memory and records of who built the Great Pyramid, as well as the knowledge of what it was for, have all but vanished. Al Manun had done extensive research and found hints in two or three ancient books that there was a secret entrance on the north side of the pyramid. There have been legends that a room inside contains treasure, nonrusting metal, bendable glass, and wonderfully accurate maps of the earth and stars.

The prince knew that the Pyramid had been standing there silently for 2½ dozen centuries. But he was unaware that visitors had found the entrance a few hundred years before his arrival (this he learned later). The entrance was invisibly closed when he arrived.

He and his men set to work. To him the huge, triangular north face of smooth, hard, white limestone

[Editor’s Note: With as much room on it as four football fields] presented a difficult search. There are 201 rows of stones, each the size of two elephants and as heavy as four elephants. The stones are finished so carefully that the seams between them almost are invisible. The stone over the entrance is balanced to pivot with the right kind of push, but who knew how to push it, even if it were found?

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Well, this search was futile, as the Prince thought it might be. Fortunately he had with him engineers (who know how to move large objects), stone masons (who know how to break and cut stone), architects, and builders. They agreed that the best thing to do was chisel their way in. But, the limestone was so hard it blunted their chisels.

So, they lit fires near the stone until it was red hot and then poured acid (vinegar) on it to make it fizz and crack. They then battered the cracked stone with poles and removed it. After they got 6 ft. in, they hit sandstone, which was softer to chisel, so they started making faster progress. They made a tunnel 100 ft. long into the Pyramid and found nothing but more stone to chisel. They were about to give up.

Suddenly, they heard something fall somewhere inside! They tunneled toward the sound and broke into a small tunnel. A few feet away was a broken stone cap with flat polished faces that looked like a coffin—with no lid. The box was red granite. There was nothing in it except a stone box at one end—the size of a normal coffin—with no lid. The box was made of highly polished, chocolate-colored granite. It was empty.

The men tore at the walls with their tools and dug a short tunnel, but left empty-handed. Instead of solving a mystery they headed home with more questions.

Al Mamun’s men went up the passage the other way 90 ft. past where they broke in and found the entrance stone from the inside. (It wasn’t in the middle of the north face but rather 24 ft. to the east of the middle of the face.) Then Al Mamun’s men went back to the plug in the ceiling and tried to chisel it out—the stone was hard quartz and mica. So they tunneled around it in the sandstone. The plug was 6 ft. long and at the end of it was . . . another plug, and after that one was . . . another one. Then there was a series of limestone plugs they broke with their chisels. The dust was suffocating. Finally they came to a 4-ft. high passageway that went uphill about 100 ft., then went level about 100 ft.

At the end of that they found a room about the size of a living room, but instead of being filled with treasure it was empty! One of the echoing stone walls had a large indentation that looked like a place for a life-sized statue, but none was there. They tunneled into the indentation but found nothing. So, they named this room the “Queen’s Room” and left.

They followed the level tunnel 100 ft. back to a large ledge above their heads that seemed to have an opening above it. When they climbed up they saw that they were in a huge hallway with a floor sloped upward like a ramp and a ceiling as high as a gymnasium (28 ft.). It had such a smooth floor that they slipped down until they found narrow ledges on each side. On these ledges were square notches near the wall every few feet. They grappled their way up these, wondering why there were polished walls with grooves running along them and why the walls bent inward to make the ceiling much smaller than the floor.

This sloped hall, which they called the “Grand Gallery,” was long. [Editor’s Note: It was half the length of a football field.] At the top end was a 3-ft. step to a blank wall with a small 3½ ft. tunnel opening at the bottom. This led shortly to an empty chamber the size of a large closet. A tunnel led out of that to a very large room twice as long as the Queen’s and with a ceiling twice as high as that of a normal room (19 ft.). This must be the treasure room!

The whole room—they called it the “King’s room”—was made of polished red granite. There was nothing in it except a stone box at one end—the size of a coffin—with no lid. The box was made of highly polished, chocolate-colored granite. It was empty.

The men tore at the walls with their tools and dug a short tunnel, but left empty-handed. Instead of solving a mystery they headed home with more questions.

Question 4: Write at least four questions that you now have about the Pyramid or its passages.

Question 5: Estimate how long it took for the men to get from the outside of the Pyramid to their disappointing look at the King’s Room. Show how you figured your estimate.
LATER HISTORY AND EXPLORATIONS OF THE GREAT PYRAMID

For about 400 years after Al Mamun’s adventure, little attention was paid to this pointed stone mountain in the desert. The Muslim religion, new in the time of Al Mamun, was flourishing by then in Egypt. Then, there was a series of earthquakes that demolished some cities of northern Egypt near the Pyramid (but not the well-built Pyramid itself.) This meant the Muslims needed a lot of building stone to rebuild their cities.

The Muslim workers climbed to the top of the Pyramid and began prying loose the highest white stones. They then let the stones slide and bounce to the sand around the bottom of the Pyramid. The stones from the Pyramid were enough to cover nine city blocks with stone about 6 ft. thick!

Many of these stones were used to rebuild the capital city of El Kaherah. Many others were used in the next hundred years to build mosques (Muslim churches) in Cairo, across the Nile River and 25 miles away. The Great Pyramid now had rubble 50 ft. up its base and rough, weathered sides. No one cared about it for hundreds of years.

Question 6: Estimate how long it would take to drag a stone weighing 20 tons (the weight of 10 pickup trucks) 25 miles with camels and horses. How did you figure your estimate?

Question 7: Give your estimate of the time it would take to drag 6,300 stones covering the Great Pyramid to Cairo.

Question 8: Estimate how many mosque building-stones could be cut from the Pyramid face-stone in the picture to the left. Assume each mosque stone is to be 2 ft. long, 1 ft. thick, and 1 ft. high.

Now, how many mosque stones could be cut from the 6,300 face stones of the Pyramid?

Starting with John Greaves of Oxford, England, in 1638, scholars have visited the Pyramid to try to understand why it was built. For instance, even Napoleon brought professors and scholars with him when his armies conquered Egypt in 1798.

They usually tried to measure everything they could—the most important being the outside measurements of the Pyramid. The mathematics and geometry of the Pyramid were believed to be the main source of clues about its meaning and purpose. Ancient books also hinted that there was something special about these measurements.

This may sound easy, but it was very difficult because there was always 50 ft. of rubble and sand covering the base. Imagine trying to measure four city blocks to the nearest foot they are covered with sand higher than the roofs of three-story houses and you might get some feeling for the job’s difficulty.

But, the great Pyramid finally has been measured in modern times quite exactly with surveying equipment and a lot of hard shoveling. The measurements have indeed shown some very interesting meanings and purposes of the Pyramid, which are brought out in *A Mathematical Mystery Tour*.

A last note: The Pyramid doesn’t stand alone on the Giza Plateau. There are two smaller pyramids and four very small ones. A little farther away stands the famous Sphinx, resembling a cross between a man and a lion. Some scholars believe it was built after the Great Pyramid. It’s about half a football field long. Legend has it that there are passages under the desert between the Pyramid and the Sphinx—something that could be checked with drilling equipment.

An obelisk (the shape of the Washington Monument) about the height of six persons once stood between the Sphinx’s paws. The obelisk would always get buried in drifting sand after explorers would uncover it. Scientists say this is a sign that the Sphinx was built before the area was a desert. That is, the intelligent Egyptians wouldn’t build something in a way that would cause the sand to constantly pile over what they built, so there must have been no sandy desert then.

Question 9: Why do you think the Sphinx faces east? Why did it have an obelisk between its paws?
The Egyptians worshiped the sun and the moon, with the sun being the greater of the two gods. The sun was born every morning from the womb of the sky, soared like a glittering falcon at midday, became an old man in the evening, and then died at night. The god Ra was the sun as it was seen at midday—proud, fierce, an all-seeing eye. At a few times in the history of the Egyptians, the sun-god was their only god.

The moon-god, Khonsu, was a lesser god but often was worshiped in Egypt’s history. Khonsu was the helper of breathing and birth because both have a cycle and rhythm, as does the monthly cycle of the moon’s phases.

On the Egyptian desert with large, open sky, both of these gods were seen playing daily. One ruled the bright day and one the dark night. Of the two, the god Ra was considered more powerful, but Khonsu was a close second.

**THE PYRAMID: ON THE GLOBE**

Find where the Great Pyramid is on the globe. It’s on the 30th degree of north latitude and the 31st degree of east longitude.

With your ruler, measure approximately how many cm of land the 30° latitude circle goes over as it goes around your globe. (Measure all of the land pieces and add them up. This is best done with a friend recording your measurements.)

Then check and see if there is any circle of latitude that seems to go over more land. Measure and see. What do you find?

Do the same for the vertical half-circle of 31° longitude. Measure all of the length of land it passes as it goes from the North Pole to the South Pole through Egypt. See if you can find another line of longitude that passes over more land. Measure and compare. What do you find?

Some people have said, “The Great Pyramid is at the center of all the land of the Earth.” Comment on this claim.
WHO WAS CHEOPS, ANYWAY?

The Great Pyramid of Cheops is the largest and most carefully made pyramid the Egyptians had. It was the first of the pointed type with straight sides—before that pyramids had large steps or changed their slant as they went up.

It took a long time and thousands of workers to make this one pyramid. The Egyptians had no wheels, but they knew how to roll stones along with logs underneath—a lot of work. They had to cut 2,300,000 stone blocks from the ground in a place called a quarry, many of which were finished and polished as perfectly as any building stones used anywhere in the 21st century.

The workers had to drag these stones weighing a few tons each on rolling logs and float them on rafts to the pyramid site and then get them up to great heights. No one knows how long all of this took, but it surely required tens of thousands of workers, some very careful organization, some brilliant architects, and some clever engineers.

But, who wanted such a big job done in the first place and who had the power and wealth to make sure it got finished? People who have studied the pyramid carefully have seemingly answered this question.

Way inside a tiny attic room above the “King’s Room” in the Pyramid, explorers found an oval stamped in red on stone and markings that said “Year 17.” They found more of these behind some limestones still in place at the base of the Pyramid. And, they also found some similar marks on stones at the quarry, ones never hauled out.

Someone who studies Egyptian history recognized the oval as the mark of Khufu, the second Pharaoh of the “Fourth Dynasty” of rulers of Egypt in about 2600 B.C. Khufu is his Egyptian name, but the ancient Greeks also had a list of Egyptian rulers that they had figured out. They called that same ruler “Cheops.” This is the name usually used with the Pyramid.

**Question 1:** How many years ago was 2600 B.C.?

Cheops’s father was Sneferu, a well-loved and hard-working ruler. The country prospered under his rule. He built about three pyramids, including the large “bent pyramid” at Daschur.

Cheops, though, was a fierce and domineering man who would have been able to control his subjects enough to get them to build a pyramid of such scale. The land was wealthy because of his father’s dedication and popularity. Much of this wealth would have gone into building the Pyramid. Cheops undoubtedly benefited from his father’s pyramid-building and hired the most brilliant architects in the land to design his own.

Cheops’s son Kephren also built a pyramid—the smaller one near the Great Pyramid. After that, pyramids continued to be built but were much more modest in size.

So, why did Khufu want such a big mountain of stone? Scholars used to think it was because he wanted to be buried in it, and called him egotistical for wanting such a giant tomb. But, as you have seen in the article “Inside the Great Pyramid,” there was no sign of Cheops’s body when the Pyramid was opened up, and no way it could have been stolen beforehand.

In fact, no Egyptian pyramid has ever been found to actually contain the body of a ruler, though in some cases grave robbers could have removed the mummified body of the pharaoh inside. The tomb of Cheops’s mother Hetepheres was found in 1925, yet the sealed coffin was empty!

This is the unanswered part of the question, then. There is no document that says why the pyramids were built. The student pages will help you suggest some reasons.

There is evidence for other reasons like observatories, ceremonial chambers, mathematical records, sundials, surveying markers, and others. The pyramids must have been a necessary part of Egyptian society and beliefs for the rulers and people to spend so much time and money building them!

**Question 2:** Why do you think Cheops wanted a pyramid built?

THE PYRAMID: AT THE CENTER

Here’s a map of where the Great Pyramid is in Egypt. Notice that the Nile River makes a big curved delta right where it flows into the Mediterranean. Set the point of your compass at the Pyramid, and the pencil at the edge of the Nile Delta. Trace an arc along the whole delta. What do you notice?
A HISTORY OF \( \pi \)

[Editor’s Note: We have put together several articles from the Time Teletype on the subject of \( \pi \). Read them all to get the history of \( \pi \).]

**Stone Age Circle Research**

**Dateline 222,264 B.C.—Kenya, Africa.** As the Tribe of the Sabre Tooth Tiger was setting up its campfire circle for this year’s fox chase ceremony, its leaders made a brilliant discovery.

They noticed two extraordinary things. First, before the fire was made, they stood a line of people shoulder to shoulder across the middle of the campfire circle. Then they stood two other lines of people close to the first line, so that all three lines crossed the circle together. The chiefs noticed that if these three lines then filed out around the edge of the campfire circle, those people just reached around it as they stood shoulder to shoulder!

So, they tried it again when the whole tribe was assembled on their big game circle. Three lines of people stood across the circle and then filed around the circle. Again, they just fit.

The medicine woman concluded that the distance around a circle is three times the distance across it, no matter what size the circle. They’re having a feast tonight to celebrate this new knowledge.

**Question 1:** How close to the true value of \( \pi \) are the tribespeople?

**Egyptian Math Goes in Circles**

**Dateline 2923 B.C.—Cairo, Egypt.**

The Mystery School of Ra has held many sessions concerning the circle. They feel that the circle is a very good picture of the infinite, as it has no end.

Their research has convinced them that if the diameter of any circle is multiplied by 64, divided by 81, and then multiplied by 4, the circumference will be accurately found.

**Question 2:** Do your own research on some circles (try starting with a circle of diameter 81) and see how close this comes to the modern value you would compute using \( \pi \). On the diameter 81 circle, give your two computed values of the circumference and their difference.

**Question 3:** (Challenge) Try to calculate what value of \( \pi \) the Egyptians’ method is equivalent to.

**Indians Declare Circle Number**

**Dateline 380 B.C.—Delhi, India.**

Hindu priests of the city have finally shared the secret truth they have known about circles for some time. In a press conference today they declared the ratio of circumference to diameter of a circle: \( 3 \frac{177}{1250} \).

It was such a shocking but beautiful truth—that all circles are essentially the same—that some people fainted. They were given herbal teas to revive them. The Hindu priests then announced a special dance and chant to their god Shiva to celebrate the truth of circles.

**Question 4:** What decimal is the value found by the priests and how close is it to the modern value of \( \pi \)?

**An Influential Greek Circle**

**Dateline 250 B.C.—Syracuse, Greece.**

Our own brilliant scientist and engineer Archimedes has unveiled his calculation of the ratio of the circumference of a circle to its diameter.

Here’s how he did it. Cleverly deciding to start with a hexagon in a circle, he easily calculated the perimeter of the hexagon, of course finding it to be less than the circumference of the circle by a fair amount. So, he broke each of the hexagon’s sides into two pieces and stretched them a bit to make a 12-sided figure in the circle that looks almost the same size as the circle itself.

He divided this answer by the diameter of the circle and proclaimed at a big Syracusan party last night that the ratio of the diameter of a circle to its circumference was between \( 3 \frac{10}{71} \) and \( 3 \frac{1}{7} \).

Archimedes feels that this discovery ranks right up there with his invention of the helical water pump and his studies of how things float. Most of you will remember that it was during his thoughts about water overflow while bathing that he got such a powerful idea that he accidentally ran through the streets with no clothes on shouting, “Eureka! I’ve found it!”

**Question 5:** How close to the modern value of \( \pi \) are Archimedes’s numbers?

**Question 6:** Draw a large circle and put points around it every 15 degrees, using a compass, protractor, and ruler. Connect these to get a 24-sided polygon in the circle. How close to actually being a circle does it look?
More History of $\pi$

Babylon Gets New Circle Law

Dateline 204 B.C.—Babylon, Middle East. Babylonian Prince Fihad has made it a law of the land that the distance around a circle shall henceforth be $3 + \left(\frac{7}{50} \cdot \frac{6}{50,000}\right)$ multiplied times the diameter of the circle.

This value shall be inscribed on stone tablets and placed in the town square. Anyone found using other values shall be forced to sit in a circle in the town square from full moon to full moon.

Question 7: What is Prince Fihad’s value for $\pi$ and how does it compare it to the modern value?

The Ultimate Polygon

Dateline 264 A.D.—City of the Gods, China. Lin Hui has just finished making a polygon in a circle. And what a polygon! It has 3,072 sides, which he got by starting with a hexagon and continuing to break the sides in half nine times to get 12, 24, 48, 96, 192, 384, 768, 1,536, and then 3,072 sides.

Then, after calculating day and night for months, he found the perimeter of that polygon to be 3.14159 times as long as its diameter. Congratulations, Lin Hui!

Question 10: How far does this value differ from the modern value to five decimal places?

Fergusen Makes Book of World Calculation Records

Dateline 1947—London, England. This September, Mr. Fergusen of mathematics fame finished computing more digits of $\pi$ than any human being in the world—808 places with a mechanical desk calculator. This breaks his previous record of 620 places set in 1946.

There had been a previous world $\pi$ record set by William Shanks in 1874—707 places, but Fergusen showed that Shanks had made a mistake at the 527th place that caused all places after to be in error. Mr. Fergusen expects that this will be the most digits known for $\pi$ for a long time because he never wants to push another button on his desk calculator with his raw, tired fingers again.

Question 9: Give Nehemiah’s value and compare it to the modern value.

ENIAC Is Maniac for $\pi$

Dateline September 1949—Ballistic Research Labs, USA. An ENIAC (which stands for Electronic Numerical Integrator and Computer), one of the first such machines ever built, has flexed its calculating muscles and found 2,037 decimal places of $\pi$ in just 70 hours. Such a feat would have taken many months of calculation by hand. The hand-calculating $\pi$ record was just reached 2 years ago by Mr. Fergusen, who is still soaking his fingers and was not totally pleased with this new breakthrough.

Chinese Find Number Buried in Circle

Dateline 130 A.D.—Shanghai, China. The Honorable Dr. Hu Fong and his associates are pleased to announce that the ratio of the circumference to the diameter of a circle has been found by them to be the number that multiplies by itself to make 10. Because 10 is such a perfect number, he feels it is only fitting that the equally perfect circle has chosen 10 in such a way.

Question 8: What is the value of $\pi$ referred to here? Experiment with your calculator to find that number. Also, try out the $\sqrt{}$ button.

Nehemiah’s Hebrew Number

Dateline 152 A.C.—Palestine. The Hebrew teacher and Rabbi Nehemiah has just published his geometry textbook with the delightful title Mishnot ha-Middot. Be sure to order 6 months ahead of time so a copy can be hand-written and sent by messenger on a donkey.

One of the most interesting points Nehemiah makes in the book is about calculating the circumference of a circle—simply multiply the diameter by $\frac{7}{3}$.

$\pi$ Chosen to Name Circle Ratio

Dateline 1706—London, England. The mathematician William Jones has just published his book called The New Introduction to Mathematics, and he hopes it will be a surefire bestseller. One of the interesting things he has done in his book is to name the circle ration number with the Greek letter $\pi$, pronounced, “pie.” For the Greeks, this makes the same sound in words as our “p.”

Mr. Jones chose this letter because the circumference is also known by the work periphery, which begins with “p.” He hopes that the very famous mathematician Leonard Euler will begin to use $\pi$ also and make it catch on among all mathematicians.

Editor’s Note: In 1737 Euler did use $\pi$ in his work and it immediately caught on with everybody from then on.]
MORE EVIDENCE FOR THE HALF-EARTH THEORY

Recall from your worksheets and model of the Great Pyramid that we have some reason to believe that the Pyramid was meant to be a scale model of half the Earth. One reason is that it has \( \pi \) in it the same way a half-sphere does.

Here’s more evidence. In half a day, 12 hours go by, which means the sun has shone over exactly one-half the Earth.

**Question 1:** How many seconds are there in a half day?

**Question 2:** Could you say that this number of seconds represents half the earth, in a way?

Now calculate how many Great Pyramids make half the Earth. Do it by finding how many times you would have to magnify the height of the Pyramid to make it as high as half the Earth, that is, the radius of the Earth.

But, what’s a suitable radius? The Earth is fatter around its equator and flatter at the poles. That’s because it’s spinning so fast. Right now you (and Mount Everest) are traveling more than 1,000 miles an hour around Earth’s outside!

This makes Earth’s 3,962-mile radius at the equator 119 miles longer than the radius at the poles. Let’s use the Earth’s radius at Egypt (1/3 of the way from equator to North Pole) and estimate it at about 3,933 miles. The Pyramid has a height of 480.7 ft.

**Question 3:** How many Pyramid heights could be placed along the 3,933 miles from the center of the Earth to where the pyramid is standing at Egypt’s surface? (Remember, there are 5,280 feet in a mile.)

**Question 4:** Compare your answer to the number of seconds representing half the Earth.

**Question 5:** Why are your two answers more reason to believe that the Egyptians wanted the Great Pyramid to represent half the Earth?

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**Believe It or Not!**

- Desert tribespeople often have reported that they see an eerie glow at the peak of the Pyramid.
- Airplane pilots are advised not to fly directly over the Pyramid as it causes instruments to read in error.
- In 1839, two explorers were working in another pyramid, that of Sneferu, Cheops’s father. It was very hot and stuffy until suddenly a cold, strong wind started in the passages and continued for 2 days. Then it stopped as unexpectedly as it started. There was no physical explanation for it.
- Sir W. Siemens, a British inventor, was standing at the top of the Great Pyramid with a friend and some Arab guides. Whenever he raised his hand with fingers spread, a sharp ringing noise was heard. Siemens could raise his finger and feel prickers in it from electricity. Upon taking a sip from the wine bottle he had with him he got a slight shock. He then had an idea.
  
  He made an electricity storage device (capacitor) by wetting a newspaper and wrapping it around the bottle. Then he held the bottle in the air above his head. When sparks started to come from this device, an Arab guide accused him of practicing witchcraft. Another guide tried to grab his companion. Siemens touched the bottle to that guide and it jolted him senseless. The guide then got up and ran down the Pyramid.

- Many who have studied the Pyramid believe it focuses energy out of its pointed top toward outer space, although there is as yet no definite scientific study of this hypothesis.

*“We have found a strange footprint on the shores of the unknown. We have devised profound theories, one after the other, to account for its origin. At last, we have succeeded in reconstructing the creature that made the footprint. And lo! It is our own.”*

—Arthur Eddington

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"We have found a strange footprint on the shores of the unknown. We have devised profound theories, one after the other, to account for its origin. At last, we have succeeded in reconstructing the creature that made the footprint. And lo! It is our own."

—Arthur Eddington
Mystery Tour Guide Answer Blank

Unit I:  
*How Liber Abaci Came to Be*, pp. 3–4

Question 1: __________________________________

Question 2: __________________________________

Question 3: (use map on pg. 3)

Question 4: __________________________________

Question 5: (use map on pg. 3)

Question 6: a. ________________________________

b. ________________________________

c. ________________________________

d. ________________________________

_Tower of Pisa Leans 5°, p. 4_

Question 1: __________________________________

Question 2: __________________________________

Unit II:  
*Bees Form Labor Union and University*, pp. 6–7

Question 1: __________________________________

Question 2: __________________________________

Question 3: __________________________________

Question 4: __________________________________

Unit III:  
*Eratosthenes Measures the Earth*, p. 8

Question 1: __________________________________

Question 2: __________________________________

Question 3: __________________________________

Question 4: __________________________________

Question 5: __________________________________

Question 6: __________________________________

Question 7: __________________________________

_Eratosthenes Measures the Earth, p. 8_

Unit III:  
*Fibonacci Discovers_, p. 9

Question 1: __________________________________

Question 2: __________________________________

Question 3: __________________________________

Question 4: __________________________________

Question 5: __________________________________

Question 6: __________________________________

Question 7: __________________________________

_Scientists Vote Ratio Most Useful Tool*, p. 10

Question 1: a. ________________________________

b. ________________________________

c. ________________________________

Question 2: __________________________________

Question 3: __________________________________

Question 4: __________________________________

Question 5: __________________________________

Question 6: __________________________________

Question 7: __________________________________

_Golden Ratio: In Style for Years*, pp. 11–12

Question 1: __________________________________

Question 2: __________________________________

Question 3: __________________________________

Question 4: __________________________________

Question 5: __________________________________

Question 6: __________________________________

Question 7: __________________________________